WIND MAPPING BY OCEAN ACOUSTIC INTERFEROMETRY

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1. INTRODUCTION

The field of surface wind blowing over the vast ocean areas is an important parameter for numerical models used for weather forecasts. Surface wind is currently measured from satellites. This provides necessary global coverage; however, measurements of high winds meet some difficulties due to weak dependency of the radar- and radiometric signals on the wind velocity at high winds. In [1] a new method of retrieval of the ocean surface wind was suggested based on measurement of the directivity of the ocean ambient noise field which to a significant extent is due to surface wind. Feasibility of such approach was demonstrated for a measuring system consisting of two ocean acoustic interferometers (each acoustic interferometer consists of two vertical line arrays (VLA) separated at a distance of the order of a few kilometers and accomplishing coherent measurement of the acoustic pressure.) In this paper somewhat different method of retrieval is suggested, and corresponding numerical simulations are performed for propagation conditions typical for North Atlantic. A time of averaging required to overcome variability of the noise field was estimated to be of the order of 3 hours. In contrast to the methods of wind retrieval based on local measurements, our approach relies on measurements of distant noise sources and it principally allows to map ocean wind over vast areas of the order of 1000 km in horizontal scale with of the order of 10 km resolution.

2. CALCULATIONS

Let us consider horizontally stratified ocean with sound speed profile and bottom properties depending on vertical coordinate *z* only and let us consider an acoustic field generated by noise sources with density $a(\vec{r},\omega)$ located at the ocean surface. Using well-known expression for the Green function we can represent the acoustic field due to this sources as follows:

$$p(\vec{r},z,t)=\int d\omega e^{-i\omega t}\int d\vec{r}\,'\, a(\vec{r}\,',\omega)\sum_n \left[-\frac{i}{4}H_0^{(1)}\left(\xi_n(\omega)\left|\vec{r}-\vec{r}\,'\right|\right)\right]u_n(z,\omega)u'_n(0,\omega) \quad (1)$$

where $u_n(z,\omega)$ is a profile of properly normalized *n*-th acoustic mode and $\xi_n(\omega)$ is a corresponding propagation constant. Density *a* is supposed to be Gaussian random field with the following correlation function:

$$\left\langle a(\vec{r},\omega)a^*(\vec{r}\,',\omega')\right\rangle = A(\vec{r})\omega^{-\gamma}\delta(\vec{r}-\vec{r}\,')\delta(\omega-\omega') \quad (2)$$

Thus, the noise sources are supposed to be stationary in time however not necessarily in horizontal plane; we have assumed additionally power-law type dependence of the sources strength on frequency. Eq. (2) is essentially Kuperman and Ingenito model [2]. We will expand also noise sources density as a superposition of some real basis functions $\varphi_k(\vec{r})$:

$$A(\vec{r})=\sum_{k=1}^{N_k}A_k\varphi_k(\vec{r}) \quad (3)$$

where A_k are *r*-independent real expansion coefficients.

We obtain an estimate of the co-spectrum of the acoustic field measured at two hydrophones with coordinates $(\vec{r}_a, z_\alpha)$ and $(\vec{r}_b, z_\beta)$ as follows:

$$C_{a\alpha,b\beta}(\omega)=\int_{-T_*/2}^{T_*/2}\frac{d\tau}{2\pi}e^{-i\omega\tau}\frac{1}{T}\int_{-T/2}^{T/2}dt\,p\left(\vec{r}_a,z_\alpha,t-\frac{\tau}{2}\right)p^*\left(\vec{r}_b,z_\beta,t+\frac{\tau}{2}\right) \tag{4}$$

Here Latin indices *a,b* correspond to a VLA and corresponding Greek indices *α,β* correspond to different hydrophones within VLA (we will assume for simplicity that VLA are strictly vertical and hydrophones on all VLA are located at the same depths.) In Eq. (4) T is an integration time used for calculation of the correlation functions and T_* is corresponding integration time used for evaluation of the co-spectrum. The results in fact do not depend on parameter T_* which is only supposed to exceed significantly the correlation time of the measured noise field; the parameter T_* is introduced in Eq. (4) to emphasize that we are dealing with signals measured within a limited period of time. Averaging Eq. (4) with account of Eqs. (1)-(3) gives:

$$\left\langle C_{a\alpha,b\beta}(\omega)\right\rangle=\omega^{-\gamma}\sum_k A_k t^k_{a\alpha,b\beta}(\omega) \tag{5}$$

where

$$t^k_{a\alpha,b\beta}(\omega)=\sum_{n,m} u_n(z_\alpha,\omega)u'_n(0,\omega)u_m(z_\beta,\omega)u'_m(0,\omega)f^k_{ab,nm}(\omega) \tag{6}$$

and

$$f^k_{ab,nm}(\omega)=\int \varphi_k(\vec{r})\left[-\frac{i}{4}H_0^{(1)}\left(\xi_n(\omega)|\vec{r}_a-\vec{r}|\right)\right]\left[-\frac{i}{4}H_0^{(1)}\left(\xi_m(\omega)|\vec{r}_b-\vec{r}|\right)\right]^* d\vec{r} \tag{7}$$

Now one tries to retrieve coefficients A_k by minimizing the mismatch between the right- and left-hand sides of Eq. (5) in an RMS sense, leading to the following linear set of equations:

$$\sum_{k'} W_{kk'}A_{k'}=U_k \tag{8}$$

where

$$U_k=\mathrm{Re}\int_{\Omega_1}^{\Omega_2} d\omega\,\omega^{-\gamma}\sum_{a\alpha,b\beta} C_{a\alpha,b\beta}(\omega)\left[t^k_{a\alpha,b\beta}(\omega)\right]^* \tag{9}$$

$$W_{kk'}=\int_{\Omega_1}^{\Omega_2} d\omega\,\omega^{-2\gamma}\sum_{n,m,n',m'} V_{nn'}(\omega)V_{mm'}(\omega)\mathrm{Re}\sum_{a,b}\left[f^k_{ab,nm}(\omega)\right]\left[f^{k'}_{ab,n'm'}(\omega)\right]^* \tag{10}$$

and

$$V_{nm}(\omega)=u'_n(0,\omega)u'_m(0,\omega)\sum_\alpha u_n(z_\alpha,\omega)u_m(z_\alpha,\omega) \tag{11}$$

In Eqs. (9),(10) (Ω_1,Ω_2) indicate the frequency interval selected for the processing. Eigenvalues of real symmetric matrix *W* are real and non-negative. If the estimate of the co-spectrum $C_{a\alpha,b\beta}(\omega)$ obtained experimentally according to Eq. (4) is close enough to its average value, a solution of Eq. (8) will provide a correspondingly accurate result.

3. SOLUTION AND INTEGRATION TIME ESTIMATE

Eq. (8) can be solved by some sort of pseudo-inversion:

$$A_k=\sum_{k'}\widetilde{W}_{kk'}U_{k'} \tag{12}$$

where matrix $\widetilde{W}$ depends on the chosen method of the pseudo-inversion; matrix $\widetilde{W}$ generally differs from W^{-1} because matrix *W* will generally have a set of small eigenvalues. Finite integration time *T* will introduce an error in the right-hand side of Eq. (12): $\delta U_k = U_k - \langle U_k \rangle$. After averaging we obtain the following expression for the corresponding correlation matrix of errors of retrieval of the amplitudes of the base functions $\delta A_k = A_k - \langle A_k \rangle$:

$$\langle \delta A_{k'} \delta A_{k''} \rangle = \sum_{k_1, k_2} \widetilde{W}_{k'k_1} \widetilde{W}_{k''k_2} \langle \delta U_{k_1} \delta U_{k_2} \rangle \tag{13}$$

To evaluate the correlator $\langle \delta U_{k_1} \delta U_{k_2} \rangle$ one has to use Eqs. (4),(9) and calculate the fourth-order correlator using Gaussianity of the source density. This is done under simplifying assumption of the set of hydrophones being dense enough so that one can take advantage of the acoustic modes orthogonality (we do not present the resulting formulae here.)

4. NUMERICAL EXAMPLE

The VLAs participating in measurements can be located sufficiently far enough from each other (say, at distances of tens of kilometers) forming an ocean interferometer (of course, all VLA of the interferometer should be synchronized and pressure signals are supposed to be recorded with respect to the same time frame.) At a given frequency the interferometer with such a large base has, of course, multiple lobes. However, using broadband signals smears out the lobes, and the interferometer will have sufficiently sharp resolution. It is clear, that sharp angular resolution still does not allow to resolve noise sources located along the same direction of observation, and to be able to map surface wind one has to have at least two interferometers observing an area of interest from different prospective (the VLAs of different interferometers do not need to be synchronized.)

We considered the case of the sound speed profile typical for the North Atlantic. The bottom model consisted of a single layer of 500 m thickness with density 1.4 g/cm³ and attenuation 0.05 dB/λ. The geometry of the numerical experiment was as follows. The area of retrieval of the wind speed was a rectangular of size 90×60 km located in the vicinity of the origin. There were two interferometers, each consisting of two VLA (four VLAs altogether). VLAs of the first interferometer had coordinates x = -310 km, y = -290 km, x = -290 km, y = -310 km; the second interferometer was a mirror image of the first with respect to x-axis. Each VLA had 50 hydrophones uniformly spread between -100 m and -4600 m depth. The base functions $\varphi_k(x, y)$ were selected to be a set of sin- and cosine spatial harmonics (12 functions altogether.) The VLAs had enough hydrophones to resolve the first 20 acoustic modes; modes of higher order were suppressed. The frequency band selected for the inversion was: $\Omega_1/(2\pi)$=10 *Hz,* $\Omega_2/(2\pi)$=20 *Hz* so that bandwidth was $\Delta F = 10\ Hz$. The ratio of the Frobenius norms of matrices $\langle \delta U_{k_1} \delta U_{k_2} \rangle$ and W was calculated to be:

$$\frac{\left\| \langle \delta U_{k_1} \delta U_{k_2} \rangle \right\|}{\left\| W \right\|^2} = \frac{1.18 \cdot 10^5}{T \Delta F} \tag{17}$$

Although for integration time $T = 10^4\ s$ the ratio of the norms is of the order of unity for 5 m/s wind, steep (close to cubic) dependence of the intensity of the noise on wind speed ensures accurate enough retrieval of strong winds. Calculation showed, that matrix W in our case has 6 sufficiently large eigenvalues (out of total 12) which were used for construction of matrix $\widetilde{W}$. The results of the retrieval are shown on the Fig. 1. One can see that the accuracy of the retrieval appeared to be of the order of 1 *m/s*. Relatively high quality of the inversion in this case is due to the fact that the original wind field was chosen close to the first eigenfunction, and admix of the higher eigenfunctions was small. If one chooses the original wind field regardless of the eigenfunctions content, the accuracy decreases to about 5 m/s which is still reasonable for high winds. We would like to emphasize, that no optimization of the design of the experimental set up was attempted, and the error can be significantly reduced by proper choice of the experimental geometry.

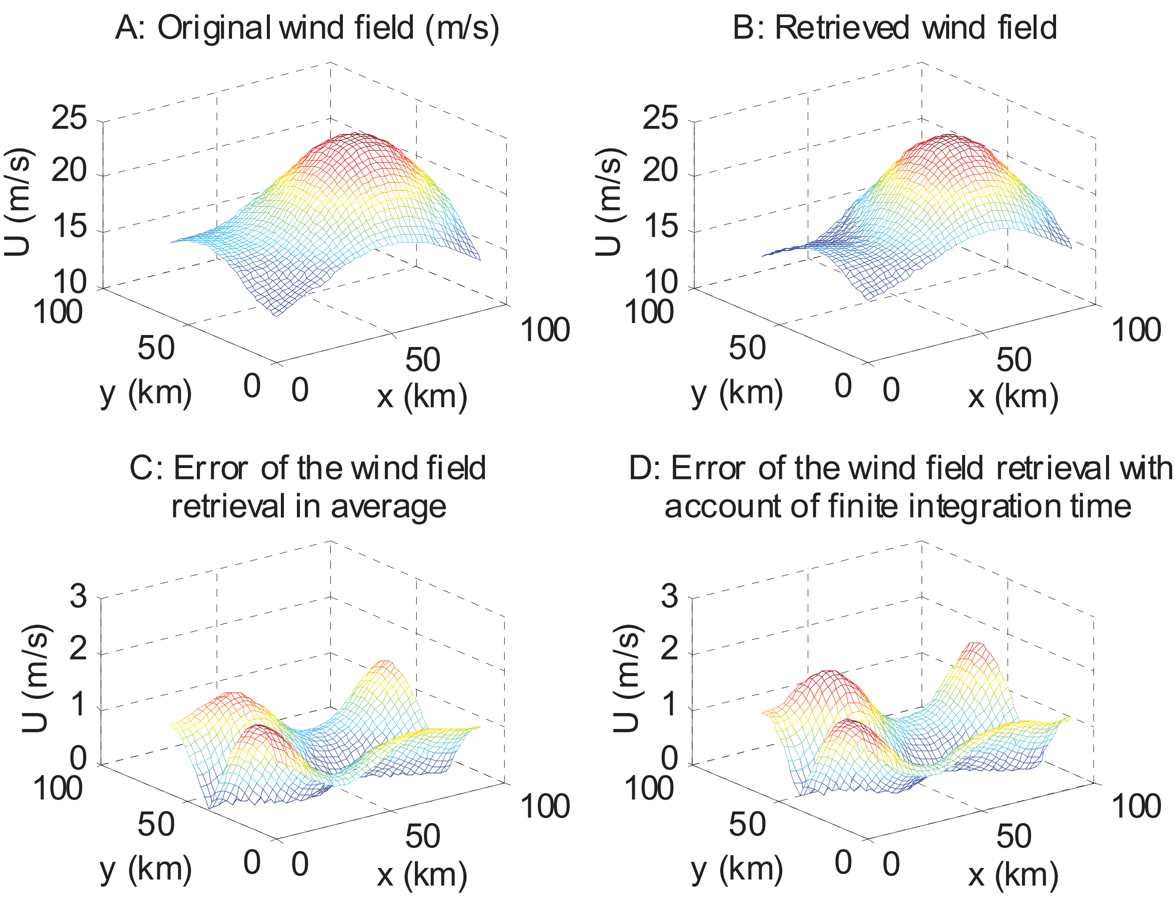

FIG. 1

The results of the retrieval of the wind field. A (upper left): an original wind field. B (upper right): the results of the retrieval in average. C (lower left): The difference between the retrieved and the original wave fields. The systematic error is due to limiting pseudo-inversion to the sub-space formed by 6 (out of total 12) eigenfunctions. The systematic error is relatively low because the original wind filed is close to the wind field due to the first eigenfunction. D (lower right): Summary error which includes both systematic error and the error due to finite integration time. The overall error is close to the systematic, due to masking effect of noise due to strong wind.

5. REFERENCES

[1] A. Voronovich, C. Penland, "Mapping ocean winds by broadband acoustic interferometry," *IGARSS 2009 (Abstracts)*, South Africa, Cape town, 2009.

[2] W.A. Kuperman, F. Ingenito, "Spatial correlation of surface generated noise in a stratified ocean," *J. Acoust. Soc. Am.,* v. 67, p. 1988-1996, 1980.