EXTENSION OF THE TARGET SCATTERING VECTOR MODEL TO THE BISTATIC CASE

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1. INTRODUCTION

In the context of Polarimetric Synthetic Aperture Radar (PolSAR) imagery, the extraction of roll-invariant parameters is one of the major point of interest for the segmentation, classification and detection. In 2007, for the monostatic case, Ridha Touzi has proposed a new Target Scattering Vector Model (TSVM) to extract physical parameters [1]. Based on the Kennaugh-Huynen decomposition, this model allows to extract four roll-invariant parameters.

For the bistatic case, the reciprocity assumption is in general no more valid. This paper presents a generalization of the TSVM when the cross-polarization terms are not equal. First, a presentation of bistatic polarimetry is exposed by means of the Kennaugh-Huynen decomposition [2]. Then, the TSVM is introduced as a projection of the scattering matrix in the Pauli basis to extract roll-invariant parameters [1] and a comparison with the monostatic case is carried out. Finally, a presentation of the computation of the TSVM parameters is exposed.

2. THE KENNAUGH-HUYNEN CON-DIAGONALIZATION

Coherent targets are fully described by their scattering matrix $\mathbf{S}$. For the context bistatic polarimetry, $\mathbf{S}$ is a complex 2×2 matrix, $\mathbf{S} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$ where the cross-polarization elements S_{HV} and S_{VH} are not equal in general.

Kennaugh and Huynen have proposed to apply the characteristic decomposition on the scattering matrix to retrieve physical parameters [2] [3] [4]. The Kennaugh-Huynen decomposition is parametrized by means of 8 independent parameters: θ_R, τ_R, θ_E, τ_E, ν, μ, κ and γ by [2] [5] [6]:

$$\mathbf{S} = e^{-j\theta_R\sigma_3}\ e^{-j\tau_R\sigma_2}\ e^{-j\nu\sigma_1}\ \mathbf{S_0}\ e^{j\nu\sigma_1}\ e^{-j\tau_E\sigma_2}\ e^{j\theta_E\sigma_3} \tag{1}$$

where:

$$\mathbf{S_0} = \mu e^{j\kappa} \begin{bmatrix} 1 & 0 \\ 0 & \tan^2\gamma \end{bmatrix} \text{ and } e^{j\alpha\sigma_k} = \sigma_0 \cos\alpha + j\sigma_k \sin\alpha. \tag{2}$$

σ_i are the spin Pauli matrices.θ_R and θ_E are the tilt angles. τ_R and τ_E are the helicity. The subscript R and E stand respectively for reception and emission. μ is the maximum amplitude return. γ and ν are respectively referred as the characteristic and skip angles. κ is the absolute phase of the target, this term is generally ignored except for interferometric applications.

Moreover, it can be shown that:

$$e^{-j\nu\sigma_1}\,\mathbf{S_0}\,e^{j\nu\sigma_1} = \begin{bmatrix} \mu e^{2j(\nu+\kappa/2)} & 0 \\ 0 & \mu\tan^2\gamma\, e^{-2j(\nu-\kappa/2)} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad (3)$$

where λ_1 et λ_2 are the two complex con-eigenvalues of $\mathbf{S}$.

3. THE TARGET SCATTERING VECTOR MODEL

3.1. Definition

The TSVM consists in the projection in the projection in the Pauli basis of the scattering matrix con- diagonalized by the Takagi method. It yields that $\mathbf{k_P} = 1/\sqrt{2}\Big[S_{HH}+S_{VV}, S_{HH}-S_{VV}, S_{HV}+S_{VH}, j(S_{HV}-S_{VH})\Big]^T$. After some mathematical manipulations, one can express the target vector $\mathbf{k_P}$ by means of Huynen's parameters by:

$$\mathbf{k_P} = \frac{1}{\sqrt{2}}\begin{bmatrix} (\lambda_1+\lambda_2)\cos(\tau_R+\tau_E)\cos(\theta_R-\theta_E)+j(\lambda_1-\lambda_2)\sin(\tau_E-\tau_R)\sin(\theta_E-\theta_R) \\ (\lambda_1-\lambda_2)\cos(\tau_R-\tau_E)\cos(\theta_R+\theta_E)+j(\lambda_1+\lambda_2)\sin(\tau_R+\tau_E)\sin(\theta_R+\theta_E) \\ (\lambda_1-\lambda_2)\cos(\tau_R-\tau_E)\sin(\theta_R+\theta_E)-j(\lambda_1+\lambda_2)\sin(\tau_R+\tau_E)\cos(\theta_R+\theta_E) \\ (\lambda_1-\lambda_2)\sin(\tau_E-\tau_R)\cos(\theta_R-\theta_E)+j(\lambda_1+\lambda_2)\cos(\tau_r+\tau_E)\sin(\theta_E-\theta_R) \end{bmatrix}. \quad (4)$$

By following the same procedure as proposed by Touzi in [1], one can introduce the symmetric scattering type magnitude and phase, denoted α_s and Φ_{α_s} by:

$$\tan(\alpha_s)\,e^{j\Phi_{\alpha_s}} = \frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}. \quad (5)$$

By combining (4) and (5), it yields:

$$\mathbf{k_P} = \mu\,e^{j\Phi_s}\begin{bmatrix} \cos\alpha_s\cos(\tau_R+\tau_E)\cos(\theta_R-\theta_E)+j\sin\alpha_s e^{j\Phi_{\alpha_s}}\sin(\tau_E-\tau_R)\sin(\theta_E-\theta_R) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}}\cos(\tau_R-\tau_E)\cos(\theta_R+\theta_E)+j\cos\alpha_s\sin(\tau_R+\tau_E)\sin(\theta_R+\theta_E) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}}\cos(\tau_R-\tau_E)\sin(\theta_R+\theta_E)-j\cos\alpha_s\sin(\tau_R+\tau_E)\cos(\theta_R+\theta_E) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}}\sin(\tau_E-\tau_R)\cos(\theta_R-\theta_E)+j\cos\alpha_s\cos(\tau_r+\tau_E)\sin(\theta_E-\theta_R) \end{bmatrix}. \quad (6)$$

Φ_s corresponds to the phase of $\lambda_1+\lambda_2$. According to (6), one can decompose $\mathbf{k_P}$ as the product of three terms:

$$\mathbf{k_P} = \mu\,e^{j\Phi_s}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_R+\theta_E) & -\sin(\theta_R+\theta_E) & 0 \\ 0 & \sin(\theta_R+\theta_E) & \cos(\theta_R+\theta_E) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos(\theta_R-\theta_E) & 0 & 0 & -\sin(\theta_R-\theta_E) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -j\sin(\theta_R-\theta_E) & 0 & 0 & -j\cos(\theta_R-\theta_E) \end{bmatrix}\begin{bmatrix} \cos\alpha_s\cos(\tau_R+\tau_E) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}}\cos(\tau_R-\tau_E) \\ -j\cos\alpha_s\sin(\tau_R+\tau_E) \\ j\sin\alpha_s e^{j\Phi_{\alpha_s}}\sin(\tau_E-\tau_R) \end{bmatrix}. \quad (7)$$

It can be noticed that the first and second terms are "rotation" matrices which depend only on the tilt angles θ_R and θ_E.

3.2. Roll-invariant target vector

As a consequence, for the bistatic case, the expression of the roll-invariant target vector $\mathbf{k_P^{orient-inv}}$ is given by:

$$\mathbf{k_P^{orient-inv}} = \mu\begin{bmatrix} \cos\alpha_s\cos(\tau_R+\tau_E) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}}\cos(\tau_R-\tau_E) \\ -j\cos\alpha_s\sin(\tau_R+\tau_E) \\ j\sin\alpha_s e^{j\Phi_{\alpha_s}}\sin(\tau_E-\tau_R) \end{bmatrix}. \quad (8)$$

In bistatic polarimetry, five parameters (namely μ, τ_R, τ_E, α_s and Φ_{α_s}) are necessary for an unambiguous description of a coherent target.

3.3. Link with the monostatic case

The monostatic case can be retrieved from the bistatic case by assuming $\theta = \theta_R = \theta_E$ and $\tau_m = \tau_R = \tau_E$. Consequently, when the reciprocity assumption holds, the roll-invariant target vector, introduced by Touzi, is:

$$\mathbf{k_P^{orient-inv}} = \mu \begin{bmatrix} \cos\alpha_s \cos(2\tau_m) \\ \sin\alpha_s e^{j\Phi_{\alpha_s}} \\ -j\cos\alpha_s \sin(2\tau_m) \\ 0 \end{bmatrix}. \quad (9)$$

4. TSVM PARAMETERS COMPUTATION

4.1. The Kennaugh matrix

The Kennaugh matrix $\mathbf{K}$ is another representation of the scattering matrix $\mathbf{S}$, its expression is given by $\mathbf{K} = 2\mathbf{A}^*\mathbf{W}\mathbf{A}^{-1}$ with $\mathbf{W} = \mathbf{S} \otimes \mathbf{S}$. $\otimes$ is the Kronecker product, and:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}. \quad (10)$$

4.2. The Kennaugh matrices of orders 0 to 2

Let $\mathbf{O_1}$, $\mathbf{O_2}$ and $\mathbf{O_3}$ be the three "rotation matrices" defined by [5]:

$$\mathbf{O_1}(2\nu) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\nu) & -\sin(2\nu) \\ 0 & 0 & \sin(2\nu) & \cos(2\nu) \end{bmatrix}, \mathbf{O_2}(2\tau) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & \sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}, \mathbf{O_3}(2\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

The Kennaugh matrices of orders 0 to 2, denoted $\mathbf{K^{(i)}}$, are defined by:

$$\begin{cases} \mathbf{K^{(2)}} = \mathbf{O_3}(-2\theta_R)\,\mathbf{K}\,\mathbf{O_3}(2\theta_E) \\ \mathbf{K^{(1)}} = \mathbf{O_2}(2\tau_R)\,\mathbf{K^{(2)}}\,\mathbf{O_2}(-2\tau_E) \\ \mathbf{K^{(0)}} = \mathbf{O_1}(-2\nu)\,\mathbf{K^{(1)}}\,\mathbf{O_1}(2\nu) \end{cases} \quad (12)$$

4.3. Link with the TSVM parameters

4.3.1. Tilt angles

In practice, thanks to the scattering scattering matrix $\mathbf{S}$, the Kennaugh matrix $\mathbf{K}$ is first computed. The tilt angles θ_E and θ_R are then directly deduced from the Kennaugh matrix $\mathbf{K}$ by [7]:

$$\tan(2\theta_E) = \frac{\mathbf{K}_{02}}{\mathbf{K}_{01}} \text{ and } \tan(2\theta_R) = \frac{\mathbf{K}_{20}}{\mathbf{K}_{10}}. \quad (13)$$

Once θ_E and θ_R are found, the Kennaugh matrix of order 2, namely $\mathbf{K^{(2)}}$, is computed according to (12). Moreover, as this matrix does not depend on the tilt angles, it can be viewed as the roll-invariant Kennaugh matrix.

4.3.2. Helicity angles

Similarly, the helicity angles τ_R are τ_E are issued from the Kennaugh matrix of order 2 by [7]:

$$\tan(2\tau_R) = \frac{\mathbf{K}^{(2)}_{30}}{\mathbf{K}^{(2)}_{10}} \text{ and } \tan(2\tau_E) = \frac{\mathbf{K}^{(2)}_{03}}{\mathbf{K}^{(2)}_{01}}. \tag{14}$$

4.3.3. ν and γ

Next, ν and γ are deduced from the Kennaugh matrices of order 1 et 0 by:

$$\tan(4\nu) = \frac{\mathbf{K}^{(1)}_{32}}{\mathbf{K}^{(1)}_{33}} \text{ and } \cos(2\gamma) = A \pm \sqrt{A^2 - 1} \tag{15}$$

with $A = \frac{\mathbf{K}^{(0)}_{11}}{\mathbf{K}^{(0)}_{01}}$. The solution adopted is the $A \pm \sqrt{A^2 - 1}$ ranging in the interval $[-1, 1]$.

4.3.4. α_s and Φ_{α_s}

Finally, the symmetric scattering type magnitude and phase, α_s and Φ_{α_s}, are directly deduced from parameters ν and γ by:

$$\tan(\alpha_s)\, e^{j\Phi_{\alpha_s}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{e^{2j\nu} - e^{-2j\nu}\tan^2\gamma}{e^{2j\nu} + e^{-2j\nu}\tan^2\gamma} = B. \tag{16}$$

It yields:

$$\tan\alpha_s = |B| \text{ and } \Phi_{\alpha_s} = \arg(B). \tag{17}$$

5. CONCLUSION

In this paper, a generalization of the Target Scattering Vector Model to the bistatic case has been proposed. Based on the Kennaugh-Huynen decomposition, five parameters are necessary for an unambiguous description of a coherent target. The ”monostatic” TSVM has been retrieved as a particular case of the proposed method. In the final version of the paper, author will present results on PolSAR data. Moreover, the roll-invariant incoherent target decomposition (ICTD) inspired from Cloude-Pottier ICTD will be introduced for the bistatic case, and a comparison with the so-called $\alpha-\beta$ model parameters will be carried out.

6. REFERENCES

[1] R. Touzi, “Target Scattering Decomposition in Terms of Roll-Invariant Target Parameters,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 45, no. 1, pp. 73–84, January 2007.

[2] J.R. Huynen, *Phenomenological Theory of Radar Targets*, Academic Press, 1978.

[3] K. Kennaugh, “Effects of Type of Polarization On Echo Characteristics,” *Ohio State Univ., Research Foundation Columbus Antenna Lab, Tech. Rep. 389-4, 381-9*, 1951.

[4] J.R. Huynen, “Measurement of the target scattering matrix,” *Proceedings of the IEEE*, vol. 53, no. 8, pp. 936–946, August 1965.

[5] A.-L. Germond, *Théorie de la Polarimétrie Radar en Bistatique*, Ph.D. thesis, Université de Nantes, Nantes, France, 1999.

[6] Z.H. Czyz, “Fundamentals of Bistatic Radar Polarimetry Using the Poincare Sphere Transformations,” Technical report, Telecommunications Research Institute, http://airex.tksc.jaxa.jp/pl/dr/20010100106/en, 2001.

[7] C. Titin-Schnaider, “Polarimetric Characterization of Bistatic Coherent Mechanisms,” *IEEE Transactions on Geoscience and Remote Sensing*, vol. 46, no. 5, pp. 1535–1546, May 2008.