SEISMIC QUALITY FACTOR ESTIMATION USING CONTINUOUS WAVELET TRANSFORM

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1. INTRODUCTION

When propagating in the Earth, seismic wave will be attenuated. Usually the seismic attenuation can be quantified by quality factor Q. The direct estimations of Q in Fourier frequency domain include logarithm spectral ratio [1], centroid frequency shift (CFS) [2], and peak frequency shift methods [3]. In these methods, before calculating its spectrum in Fourier domain, the recording will be truncated by a time window. If the time window is inappropriate, the estimated Q may deviate more from accurate value. Based on Gabor-Morlet transform, the Q estimation [4] can avoid the problem of time window. By continuous wavelet transform (CWT), the relationship between peak scale of the scalogram in wavelet domain and Q is derived [5], and it can be used to estimate Q accurately. However, the relationship is based on the assumption of impulse source wavelet.

In this paper, we assume source being a constant-phase and develop a formula of frequency-independent Q estimation by the ratio of wavelet-domain peak amplitude. And Morlet wavelet is selected as mother wavelet in CWT. Finally, we test our formula by synthetic and real zero-offset VSP data.

2. METHODOLOGY

If source wavelet is constant-phase, its corresponding frequency-domain expression can be written as

$$U(\omega;0)=\exp[-(\omega-\sigma_0)^2/(2\delta_0{}^2)+i\varphi_0],\tag{1}$$

where, σ_0 and δ_0 denote the dominant (angular) frequency and standard deviation, respectively, and phase φ_0 is a constant. In real VSP data test, σ_0, δ_0 and φ_0 in (1) are usually unknown and should be estimated previously.

When the source wavelet propagating in the anelastic media with frequency-independent Q, the frequency-domain signal can be expressed as

$$U(\omega;t)=U(\omega;0)\exp\left[-i\omega t-\omega t/(2Q)\right],\tag{2}$$

where, $\mathrm{i}=\sqrt{-1}$, ω is angular frequency, t is travel time. Performing CWT in frequency domain on $U(\omega;t)$ gives [6]

$$W[\psi;U](b,a)=[1/(2\pi)]\cdot\int_{-\infty}^{\infty}U(\omega;t)[\sqrt{a}\hat{\psi}(a\omega)\exp(-i\omega b)]^{*}d\omega\ ,\tag{3}$$

where, b, a are translation and scale factor respectively, $\sqrt{a}\hat{\psi}(a\omega)\exp(-i\omega b)$ denotes the Fourier transform of wavelet function family $\psi((t-b)/a)/\sqrt{a}$, and the asterisk means the conjugate. If Morlet wavelet is selected, $\hat{\psi}(a\omega)$ is rewritten as

$$\hat{\psi}(a\omega)=\exp[-(a\omega-\sigma)^2]\ ,\tag{4}$$

where, σ is modulated frequency. And in this paper, we select σ as 6.0. Substituting of (1) and (4) into (3) and with the fixed scale a, then let $b = t$, we will obtain the wavelet-domain peak amplitude $|W[\psi;U](a)|_{\max}$ of $W[\psi;U](b,a)$ [5]

$$|W[\psi;U](a)|_{\max} = \sqrt{a} \cdot \exp\{-[\sigma_0^{\ 2}/(2\delta_0^{\ 2}) + \sigma^2]\} \cdot \exp\{\frac{[\sigma_0/(2\delta_0^{\ 2}) - t/(4Q) + a\sigma]^2}{[1/(2\delta_0^{\ 2}) + a^2]}\} \Big/ \sqrt{4\pi[1/(2\delta_0^{\ 2}) + a^2]}. \tag{5}$$

Suppose the reference and target recorded wavelets are $U_1(\omega;t_1)$ and $U_2(\omega;t_2)$, respectively, and the corresponding travel times are t_1 and t_2 ($t_2 > t_1$); in terms of (5), the ratio of wavelet-domain peak amplitude of these two recordings generates

$$\frac{|W[\psi;U_2](a)|_{\max}}{|W[\psi;U_1](a)|_{\max}} = \exp\{\frac{-\Delta t \cdot [(\sigma_0/\delta_0^2 + 2a\sigma) - (t_1/(2Q) + \Delta t/(4Q))]}{4Q \cdot [1/(2\delta_0^2) + a^2]}\} , \tag{6}$$

where, travel time interval $\Delta t = t_2 - t_1$. Usually Q value is more than 10.0 and the value of t_1, Δt are considerably small, so the exponential term $t_1/(2Q) + \Delta t/(4Q)$ in the right side of (6) will meet $t_1/(2Q) + \Delta t/(4Q) << \sigma_0/\delta_0^{\ 2} + 2a\sigma$ and can be omitted here. And then we changes (6) into

$$\frac{|W[\psi;U_2](a)|_{\max}}{|W[\psi;U_1](a)|_{\max}} = \exp\{\frac{-\Delta t \cdot (\sigma_0/\delta_0^{\ 2} + 2a\sigma)}{4Q \cdot [1/(2\delta_0^{\ 2}) + a^2]}\} , \tag{7}$$

taking natural logs of the two sides of (7) gives

$$\ln(|W[\psi;U_2](a)|_{\max} / |W[\psi;U_1](a)|_{\max}) = \frac{-\Delta t \cdot [\sigma_0/(2\delta_0^{\ 2}) + a\sigma]}{2Q \cdot [1/(2\delta_0^{\ 2}) + a^2]} ; \tag{8}$$

then make the variable $\eta = [\sigma_0/(2\delta_0^{\ 2}) + a\sigma)]/[1/(2\delta_0^{\ 2}) + a^2]$, (8) will be rewritten as

$$-\ln(|W[\psi;U_2](a)|_{\max} / |W[\psi;U_1](a)|_{\max}) = \Delta t \cdot \eta/2Q. \tag{9}$$

If the standard deviation δ_0 approaches the infinity, (9) will be changed to

$$-\ln(|W[\psi;U_2](a)|_{\max} / |W[\psi;U_1](a)|_{\max}) = \Delta t \cdot \sigma/(2aQ) , \tag{10}$$

and (10) is the Q estimation formula when the source wavelet is an impulse.

In this paper, we discretize the scale factor a as a set of scales $a = a_0 2^{-(j-1)\times 0.125}$, $j = 1, \cdots, J$, where a_0 and J are the largest scale and the number of discrete scales, respectively. And a_0 and J are determined by the usable bandwidth of source wavelet. In terms of (9), by plotting $-\ln(|W[\psi;U_2](a)|_{\max} / |W[\psi;U_1](a)|_{\max})$ as a function of variable $\Delta t \cdot \eta/2$ and fitting a straight line using least-square linear regression, Q can be obtained from the slope of the fitted line.

3. EXAMPLES

3.1. Synthetic Zero-offset VSP Data

We calculate the synthetic zero-offset VSP data with the dominant frequency and standard deviation of constant-phase source wavelet set as $\sigma_0 = 60\pi$ and $\delta_0 = 75$, respectively, and for simplicity, phase as $\varphi_0 = 0$. We chose the effective frequency range of recordings as 15-80Hz; then the maximum and minimum of scale factor a are evaluated as 0.064 and 0.012, and $J = 19$. In Fig. 1(a), the depth interval between two adjacent geophones is 20 m and the depth range is 400 m to1600 m. In Fig. 1(b) only the direct wave is considered and the sampling rate is 1 KHz. We display the Q estimation of the first and second layer by least-square linear regression, in terms of (9) and (10), respectively, in Fig. 2 and 3. In the first layer the Q values are obtained as 105.0 and 139.5. In the second layer, the estimated Q value is 52.0 and 69.3, respectively. Comparing with the exact Q values given in Fig. 1(a), we can find that when the source is modeled by a constant-phase wavelet in (1), the estimated Q value by (9) will be more accurate than that by (10) of impulse source wavelet.

3.2. Real Zero-offset VSP Data

The real zero-offset VSP data are gathered in the Sulige gas field of the northwest district of China. Tthe geophones are located with 5 m interval. Fig. 4(a) shows the recordings with sampling frequency being 1KHz. By the classic Newton iteration method, σ_0 and δ_0 are estimated as $\sigma_0 = 60\pi$, $\delta_0 = 70$. As σ_0, δ_0 approach to that of source wavelet in synthetic example, we chose the same a as that used in the synthetic. In terms of velocity and geological profile, the depth interval can be subdivided into five layers approximately. Within each layer, we estimate Q from different recording pairs comparison by (9), (10) and CFS method [4] respectively, and then we regard the median of obtained reasonable Q values, i.e. positive value, as the corresponding layer- Q. The final layer- Q is shown in Fig. 4(b) and the gray circles indicate the distribution of gas reservoirs. We can notice that Q value estimated by (9) (the solid line) is very similar to that estimated by CFS (the solid line marked by crossings), however, Q values estimated by (10) (the solid line marked by boxes) deviate more. In the fourth layer of gas accumulation, Q value estimated by (9) is approximately 30, which is lower and very near to the estimated Q values in [7].

4. DISCUSSION AND CONCLUSION

With the assumption of constant-phase source wavelet, based on the one-way wave propagation theory in anelastic media and CWT, the formula of frequency-independent Q estimation in wavelet domain is derived. The examples of synthetic zero-offset VSP data demonstrate that when the source is constant-phase wavelet, the estimated Q value by using our formula is nearer to the true value than that by the formula of impulse source. The results of real zero-offset VSP indicate that our method provides more accurate Q information than that based on impulse source. And also the corresponding layer- Q is consistent with well log, which may aid in the interpretation of gas reservoirs characteristics and lithological discrimination. Although the variation of Q value estimated by (10) is similar to that of Q estimated by our formula, Q value estimated by (10) is unreliable, which may cause appreciable deviation in inverse-Q filtering.

5. ACKNOWLEDGEMENT

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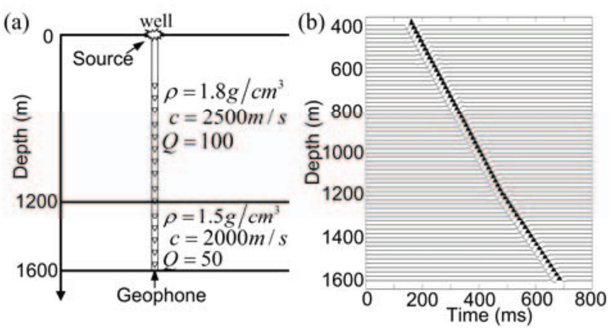

Fig.1 The synthetic zero-offset VSP data: (a) two-layer depth model; (b) the synthetic VSP section.

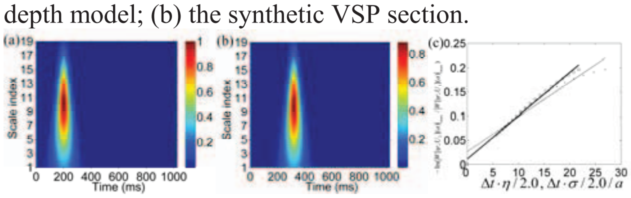

Fig. 2 *Q* estimation in the first layer of the synthetic VSP: (a) and (b) are the wavelet-domain amplitude spectrum $|W[\psi;U_1](a)|$, $|W[\psi;U_2](a)|$ of VSP recording at depth of 500 m and 800 m respectively; (c) the natural logs of wavelet-domain peak amplitude ratio. In Fig. 2(c) and in the following Fig. 3(c), thicker and thinner black lines are the fitted lines corresponding to (9) and (10), respectively.

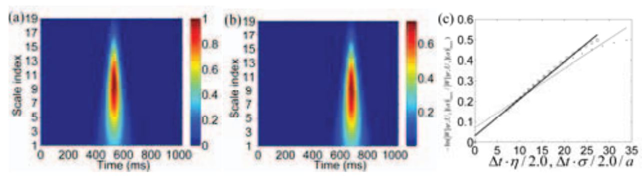

Fig. 3 *Q* estimation in the second layer of the synthetic VSP: (a) and (b) are the wavelet-domain amplitude spectrum of VSP recording at depth of 1300 m and 1600 m, respectively; (c) the natural logs of wavelet-domain peak amplitude ratio.

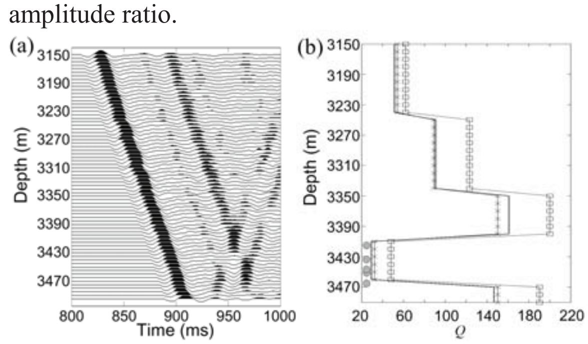

Fig. 4 the *Q* estimation of real zero-offset VSP data: (a) the real zero-offset VSP recordings; (b) the estimated layer-*Q* value by (9)(solid line), (10)(solid line marked by boxes), and CFS (solid line marked by crossings) respectively, and the gray circles denote the gas reservoirs.