CHARACTERISTICS OF ROUGH SURFACE PARAMETERS ESTIMATED FROM MEASURED SURFACE PROFILE OF FINITE LENGTH

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1. INTRODUCTION

Rough surface parameter estimation is one of the important problems in the field of radar remote sensing that retrieve geophysical parameters such as soil roughness, moisture, and dielectric constant form synthetic aperture radar data. When we estimate roughness parameters such as root-mean-square (rms) height and correlation length from a surface height-profile measured by scanning devices, data samples with sufficiently long record length are necessary [1][2] for accurate estimation. However, the criterion of the data length required for accurate estimation of these parameters is not clear. In our previous study [3][4], we revealed a relationship between the data record length and accuracy of estimated parameters in analytical form. In the study, we assumed that mean and trend (or bias and inclination angle) of the measured surface height-profile were known. However, in an actual situation, these values are unknown because a base line of the rough surface is unknown and thus they should be estimated and removed from the measured data. In this study, we present a method for estimation of the bias and the inclination from surface profile data and reveal the accuracy of them.

2. ESTIMATION OF ROUGHNESS PARAMETERS

We consider a one-dimensional Gaussian random rough surface whose profile is described by a height function $z = f(x)$ as shown in Fig. 1, and assume that its statistical properties are invariant under the translation of spatial coordinate x (stationary). In our previous study, we calculated the rms-height and correlation length using the surface profile $f(x)$ of finite length L and evaluated the errors of parameter estimation arising from finite data length under the assumption that $f(x)$ is the Gaussian distribution function with mean value zero. In an actual situation, however, it is difficult to measure the surface profile $f(x)$ in pure form by using profiling instruments such as laser profilers, because a base line of the surface (x-axis in Fig. 1) is unknown. As shown in Fig. 1, the data obtained from the measurement along the measurement line $(z = ax + b)$ is only a distance $g(x)$ between the measurement line and the soil surface. The distance $g(x)$ is expressed as

$$g(x) = ax + b - f(x) \tag{1}$$

where a and b are constants correspond to the inclination and the bias of the measurement line. In order to determine the base line (x-axis), we first determine the unknown constants a and b using $g(x)$ $(0 \le x \le L)$ in the sense of least square. Since a mean value of $f(x)$ is zero $(\langle f(x) \rangle = 0)$, we define the following mean square error

$$\varepsilon_L = \frac{1}{L}\int_0^L [g(x) - (ax+b)]^2 \, dx \tag{2}$$

and determine a and b so that the error ε_L becomes minimum by applying the least square method. As a result, we have

$$a_L = \frac{6}{L}(2I_{g1} - I_{g0}), \quad b_L = 4I_{g0} - 6I_{g1} \tag{3}$$

where I_{g0} and I_{g1} are integrals of measured data defined by

$$I_{g0} = \frac{1}{L}\int_0^L g(x)dx, \quad I_{g1} = \frac{1}{L^2}\int_0^L xg(x)dx \tag{4}$$

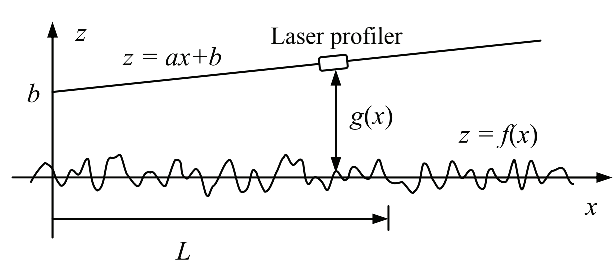

Fig. 1: Rough surface with surface height-profile $f(x)$ and measurement of it using a laser profiler along a measurement line $z = ax + b$.

Thus, the surface height-profile is expressed in terms of these constants as follows:

$$f_L(x) = a_L x + b_L - g(x) \tag{5}$$

For sufficiently long measured data $(L \to \infty)$, we can show

$$\lim_{L\to\infty} a_L = a, \quad \lim_{L\to\infty} b_L = b, \quad \text{and} \quad \lim_{L\to\infty} f_L(x) = f(x) \tag{6}$$

Using the surface profile $f_L(x)$ obtained above, the rms-height h, auto-correlation function $C(x)$, and correlation length l_c of the surface can be estimated using the following equations:

$$\hat{h}^2 = \frac{1}{L}\int_0^L f_L^2(x)dx, \qquad \hat{C}(x) = \frac{1}{L}\int_0^L f_L(x')f_L(x'+x)dx', \qquad \frac{\hat{C}(\hat{l}_c)}{\hat{h}^2} = \frac{1}{e} \tag{7}$$

where the mark ^ denotes an estimator. In order to evaluate accuracy of these estimates, we calculate means of these roughness parameters. As the final results, we can obtain the following expression for $\hat{h}^2$ and $\hat{C}(x)$:

$$\left\langle \hat{h}^2 \right\rangle = h^2 - \frac{4}{L}\int_0^L C(x)dx, \qquad \left\langle \hat{C}(x) \right\rangle = C(x) - \frac{4}{L}\int_0^L C(x)dx \tag{8}$$

where $\langle\cdot\rangle$ indicates ensemble averaging operation. As the important special case, we here consider the Gaussian correlation function given by

$$C(x) = h^2 \exp\left(-x^2/l_c^2\right) \tag{9}$$

Thus, we have the following results:

$$\left\langle \hat{h}^2 \right\rangle = h^2 - \frac{2\sqrt{\pi}\,h^2 l_c}{L}, \qquad \left\langle \hat{C}(x) \right\rangle = C(x) - \frac{2\sqrt{\pi}\,l_c}{L}h^2 \tag{10}$$

Using the above results, we can also obtain the mean of the correlation length estimate as follows:

$$\left\langle \hat{l}_c \right\rangle = l_c - \frac{(e-1)\sqrt{\pi}\,l_c}{L} \tag{11}$$

These results indicate that both the estimates $\hat{h}^2$ and $\hat{l}_c$ obtained Eq. (7) give underestimated values and they approach to the true values h^2 and l_c as the data record length L increases. This fact agrees with simulated results [1].

It should be noted that Eqs. (10) and (11) give statistical behavior of the estimates in analytical form, and thus, they give us information for correcting the estimation errors. After recursive calculation, expressions of corrected parameters can be obtained as follows:

$$\tilde{h}^2 = \left(1 - \frac{2\sqrt{\pi}\,l_c}{L}\right)^{-1}\hat{h}^2, \qquad \tilde{l}_c = \hat{l}_c + \frac{(e-1)\sqrt{\pi}\,\hat{l}_c^2}{L} \tag{12}$$

By applying this correction, we can reduce the errors caused by finiteness of the sample length and can obtain more accurate estimates. For exponential correlation type, similar results can be obtained.

4. REFERENCES

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