ESTIMATION OF THE DEGREE OF POLARIZATION IN COMPACT POLARIMETRY

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1. INTRODUCTION

Throughout the years and especially in the last two decades, polarimetric imagery has been significantly developed as a powerful tool providing complementary information in a variety of fields. Polarimetric imagery systems are now used in a wide range of applications such as meteorological investigations [1], estimation of forest parameters [2], astronomy [3], computer vision [4], and medicine [5].

Radar polarimetry has been the subject of increasingly extensive international research since the release of the first polarimetric radar concepts in the 1950s. Early synthetic aperture radar (SAR) systems, single-pol systems, were designed using a single polarization; transmitting and receiving horizontally (H-H), or vertically (V-V), polarized radiation. More sophisticated polarimetric systems are full polarimetric (FP) systems, quad-pol systems, which alternately transmit two orthogonal polarizations and record both received polarizations (H-H, H-V, V-H, V-V). Full polarimetric systems allow much more information to be extracted from a scene. However, they suffer from a lower radar swath coverage, and higher antenna transmitter power requirements. As a result, dual-polarization (DP) systems have been widely investigated in recent years as a possible tradeoff in polarimetric SAR. In a dual-pol mode, only one polarization is transmitted whereas two are received (H-H, H-V) or (V-H, V-V). Recent interest in dual-pol systems has led to a novel approach called partial polarimetry or compact polarimetry (CP) imaging. CP provides more information than a single-pol system, while not suffering as much from the drawbacks of a quad-pol system. Souyris et al. [6] introduced the $\pi/4$ compact polarimetric mode, where the transmitted polarization is the superposition of linear horizontal and vertical polarizations (H+V oriented at $45°$) and the received returns are recorded in both horizontal and vertical polarizations (H+V-H, H+V-V). In another study, Stacy and Preiss [7] proposed the dual circular compact polarimetry (DCP) mode based on a right circular polarization transmit, and right and left circular polarization receives (R-R, R-L). In a recent study, Raney [8] suggested a mode of operation, called hybrid mode or CTLR, with a circular polarization on transmission and two linear polarizations on reception (R-H, R-V). Analysis of dual-pol SAR imagery has gained new importance with the recent launches of the ALOS PALSAR, RADARSAT-2, and TerraSAR-X polarimetric SAR systems, in which a majority of the collected data is restricted to dual-pol imaging modes. Therefore, it is important to understand the information content of these images and appreciate the suitable CP configurations to use for particular applications. In fact, different CP modes collect different aspects of the information content of fully polarimetric data, and thus, comparison between these modes are not always straightforward.

In this paper, we generalize the maximum likelihood estimators (MLE) of the degree of polarization (DoP), proposed in [9] for optical polarimetric imagery, to SAR compact polarimetry. The DoP is a scalar parameter commonly used to characterize the polarization properties of a scene. Knowledge of DoP helps to gain a better understanding of the wave and scattering medium interaction, and thus, a better analysis and interpretation of the acquired data. We use a full quad-pol data set, acquired by RADARSAT-2, in order to generate different CP modes and study the performance of DoP MLEs in these modes.

2. MAXIMUM-LIKELIHOOD ESTIMATION OF POLARIZATION DEGREE

An electromagnetic radiation consists of two components: an electric field and a magnetic field. The polarization properties of electromagnetic waves can be described by their electric field. The covariance matrix of the electric vector $\boldsymbol{E} = (E_x, E_y)^T$ is defined as

$$\boldsymbol{\Gamma} = \mathbb{E}\left[\boldsymbol{E}\boldsymbol{E}^{\dagger}\right] \triangleq \begin{pmatrix} a_1 & a_3 + ia_4 \\ a_3 - ia_4 & a_2 \end{pmatrix}$$

where $\mathbb{E}[\cdot]$ is the expectation operator, and † the conjugate transpose. The state of the polarization of an electromagnetic wave can be characterized by the square DoP defined as [10, p. 134-136]

$$\mathcal{P}^2 = 1 - 4\frac{|\mathbf{\Gamma}|}{[\text{trace}(\mathbf{\Gamma})]^2} = 1 - \frac{4\left[a_1 a_2 - (a_3^2 + a_4^2)\right]}{(a_1 + a_2)^2}. \quad (1)$$

Note that the wave is totally depolarized for $\mathcal{P} = 0$, totally polarized for $\mathcal{P} = 1$, and partially polarized when $\mathcal{P} \in]0, 1[$. For a given incident polarization, the Stokes parameters of the backscattered field, expressed in linear or circular polarization bases, are given by [11]

$$\begin{aligned} I &= \left\langle |E_\text{H}|^2 + |E_\text{V}|^2 \right\rangle = \left\langle |E_\text{L}|^2 + |E_\text{R}|^2 \right\rangle \\ Q &= \left\langle |E_\text{H}|^2 - |E_\text{V}|^2 \right\rangle = 2\Re \left\langle E_\text{L} E_\text{R}^* \right\rangle \\ U &= 2\Re \left\langle E_\text{H} E_\text{V}^* \right\rangle = 2\Im \left\langle E_\text{L} E_\text{R}^* \right\rangle \\ V &= 2\Im \left\langle E_\text{H} E_\text{V}^* \right\rangle = \left\langle |E_\text{L}|^2 - |E_\text{R}|^2 \right\rangle \end{aligned}$$

where E denotes the complex electric field received in the subscripted polarization, * denotes complex conjugate, $\langle\cdot\rangle$ denotes ensemble averaging, $\Re(z)$ and $\Im(z)$ denote the real and imaginary components of z respectively.

Several useful quantitative measures follow from the Stokes formalism for SAR data. In particular, the DoP, equivalently defined using Stokes parameters as $\mathcal{P} = \left(Q^2 + U^2 + V^2\right)^{1/2}/I$, the degree of linear polarization $\mathcal{P}_\text{L} = \left(Q^2 + U^2\right)^{1/2}/I$, the degree of circular polarization $\mathcal{P}_\text{C} = V/I$, the circular polarization ratio $\mu_\text{C} = (I - V)/(I + V)$, and the linear polarization ratio $\mu_\text{L} = (I - Q)/(I + Q)$. However, images conventionally delivered by traditional dual-pol radars are only sufficient for calculating two of the Stokes parameters: (I, Q) in linearly polarized, and (I, V) in circularly polarized case. As a consequence, potentially valuable information are removed, and the parameters cited above, which depend on the complex cross product between channels, cannot be evaluated straightforwardly. The focus of this paper is, therefore, the estimation of the DoP using only two intensity images and the evaluation of the performance of DoP MLEs in different CP modes. To that end, we briefly study the MLE of DoP in what follows. The estimation of $\mathcal{P}^2$ from expression (1) can be conducted by estimating the parameters of the covariance matrix, i.e., a_1, a_2, and $r = a_3^2 + a_4^2$.

It is well known that, assuming fully developed speckle, the Jones vector $\boldsymbol{E} = (E_x, E_y)^T$ is distributed according to a complex Gaussian distribution [10, 12] whose probability density function is

$$p(\boldsymbol{E}) = \frac{1}{\pi^2 |\mathbf{\Gamma}|} \exp\left(-\boldsymbol{E}^\dagger \mathbf{\Gamma}^{-1} \boldsymbol{E}\right).$$

Let us consider the random Hermitian matrix $\mathbf{A}_{\boldsymbol{E}}$, corresponding to the q-look intensity images, expressed as

$$\mathbf{A}_{\boldsymbol{E}} = \left(\frac{1}{q}\sum_{j=1}^{q} \boldsymbol{E}_j \boldsymbol{E}_j^\dagger\right) \times q = \sum_{j=1}^{q} \boldsymbol{E}_j \boldsymbol{E}_j^\dagger.$$

The random Hermitian matrix $\mathbf{A}_{\boldsymbol{E}}$ is distributed according to a Wishart distribution [13, th. 5.1]. As a consequence, the distribution of the multi-look intensity vector $\boldsymbol{I} = (I_1, I_2)^T$, where $I_1 = |E_x|^2$ and $I_2 = |E_y|^2$, is a bivariate gamma distribution (BGD) [14, 15] parameterized by a_1, a_2, and $r = a_3^2 + a_4^2$. Therefore, based on the BGD distribution and adopting the method proposed in [15], these three parameters can be estimated using the MLE principle. The MLEs of a_1 and a_2 are expressed as

$$\widehat{a}_l = \frac{1}{qn}\sum_{j=1}^{n} I_l^j \qquad l = 1, 2 \quad (2)$$

where n is the number of samples. In practice, $\widehat{a}_l$ is calculated for each pixel by using a sliding square window (centered on the considered pixel) and computing the empirical mean over the n pixels (denoted as I_l^j) contained in the window. Moreover, the MLE of r, denoted as $\widehat{r}$, satisfies the following nonlinear relation (more details are provided in the final paper)

$$\widehat{a}_1\widehat{a}_2 - \widehat{r} - \frac{1}{qn}\sum_{j=1}^{n} I_1^j I_2^j \frac{f_{q+1}\left(\widehat{c} I_1^j I_2^j\right)}{f_q\left(\widehat{c} I_1^j I_2^j\right)} = 0$$

where $\widehat{c} = \widehat{r}/(\widehat{a}_1\widehat{a}_2 - \widehat{r})^2$, and $f_q(z) = \sum_{j=0}^{\infty} z^j / (\Gamma(q + j) j!)$ is related to the confluent hypergeometric function [16, p. 374]. The practical determination of $\widehat{r}$ is achieved by using a Newton-Raphson procedure. The MLEs of a_1, a_2, and r are then plugged into (1), yielding a DoP estimate based on two intensity images, i.e., $\widehat{\mathcal{P}}^2 = 1 - 4\left(\widehat{a}_1\widehat{a}_2 - \widehat{r}\right)/(\widehat{a}_1 + \widehat{a}_2)^2$.

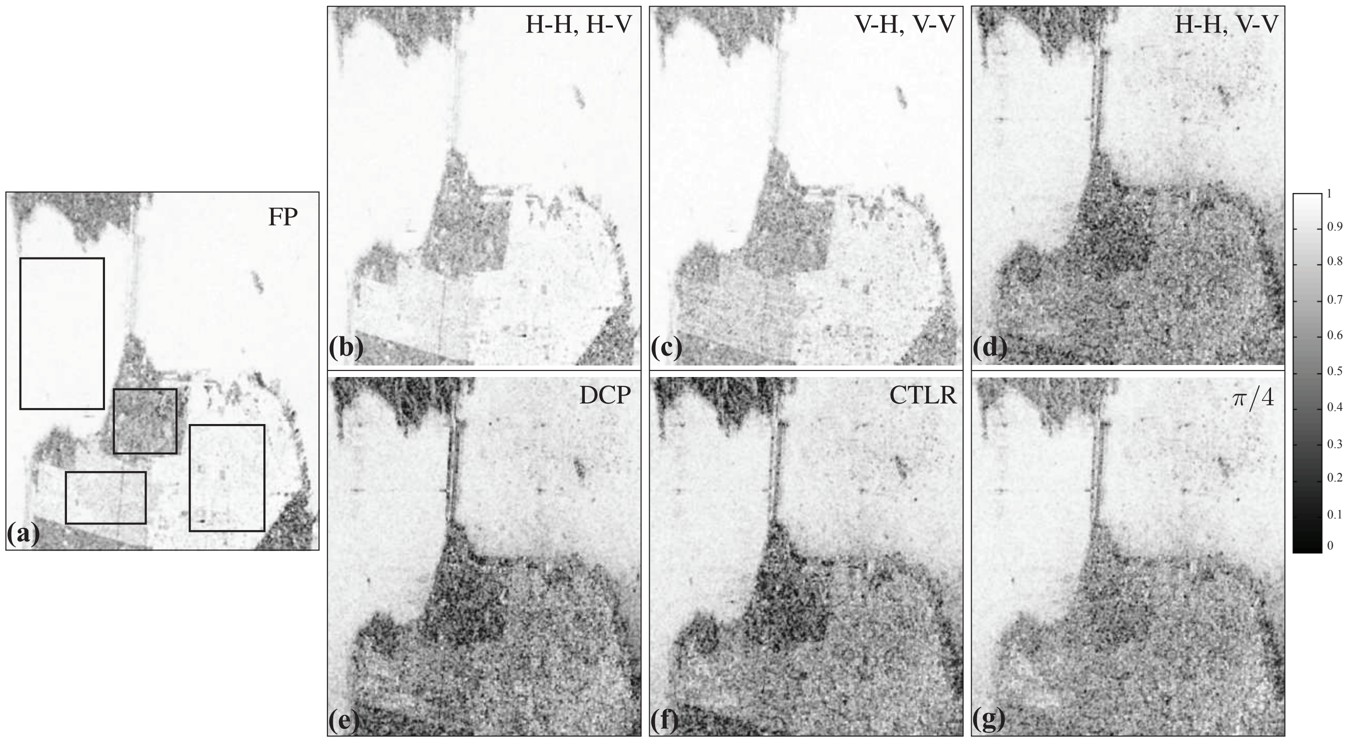

Fig. 1. Maps of the degree of polarization, in full and compact polarimetric modes, over San Francisco Bay area. (a) Full polarimetric mode (used as the reference). (b) H-H, H-V. (c) V-H, V-V. (d) H-H, V-V. (e) DCP. (f) CTLR. (g) $\pi/4$. The four outlined areas in (a) are (from left to right) ocean, urban 1, park, and urban 2 regions. The original image is single-look with 1270 pixels in azimuth and 1450 pixels in range. A sliding window covering 9×9 pixels is used.

3. COMPARISON OF DOP ESTIMATIONS IN DIFFERENT COMPACT POLARIMETRY MODES

To compare the MLEs of DoP in different CP modes, a full quad-pol data set was used to generate different CP modes. The scattering vector $\vec{k} = [S_{\mathrm{HH}}, S_{\mathrm{HV}}, S_{\mathrm{VH}}, S_{\mathrm{VV}}]^T$ represents the quad-pol SAR data set. Note that under the assumption of scattering reciprocity, the S_{HV} and S_{VH} elements are equal. The scattering vectors for the $\pi/4$, dual circular polarimetric (DCP), and right circular transmit, linear (horizontal and vertical) receive (CTLR) modes are given by [6, 7, 8]

$$\begin{aligned} \vec{k}_{\pi/4} &= [S_{\mathrm{HH}} + S_{\mathrm{HV}}, S_{\mathrm{VV}} + S_{\mathrm{HV}}]^T / \sqrt{2} \\ \vec{k}_{\mathrm{DCP}} &= [S_{\mathrm{RR}}, S_{\mathrm{RL}}]^T \\ &= [(S_{\mathrm{HH}} - S_{\mathrm{VV}} + i2S_{\mathrm{HV}}), i(S_{\mathrm{HH}} + S_{\mathrm{VV}})]^T / 2 \\ \vec{k}_{\mathrm{CTLR}} &= [S_{\mathrm{HH}} - iS_{\mathrm{HV}}, -iS_{\mathrm{VV}} + S_{\mathrm{HV}}]^T / \sqrt{2}. \end{aligned}$$

The scattering vectors for traditional dual-pol modes are given by $\vec{k}_1 = [S_{\mathrm{HH}}, S_{\mathrm{HV}}]^T$, and $\vec{k}_2 = [S_{\mathrm{VH}}, S_{\mathrm{VV}}]^T$. Moreover, for comparison purpose, we introduce $\vec{k}_3 = [S_{\mathrm{HH}}, S_{\mathrm{VV}}]^T$. Hence, in each mode, the two delivered intensity images are related to the elements of the corresponding scattering vector by $I_1 = |k_1|^2$, and $I_2 = |k_2|^2$. In this paper, we consider a RADARSAT-2 single-look fully polarimetric image[1] of San Francisco Bay in California, as a test case to evaluate the DoP MLEs. This data set consists of three main regions: ocean, man-made structures (urban area, the bridge, etc.), and the vegetated area (park). Therefore, the utility of DoP estimation for different CP modes can be compared over these different terrain types.

Figure 1 shows the DoP MLEs obtained using two intensity images delivered in each compact polarimetry mode. It shows that the MLEs have a similar global behavior with the ocean region having an average DoP close to 0.97 and the vegetated area having an average DoP close to 0.70 (more details are provided in the final paper). We note that DoP estimations based on dual-pol data reveal two different urban areas in Fig. 1(a) and Fig. 1(b). A visual inspection of the results suggests that

[1] Avaliable online: `http://www.radarsat2.info`

traditional dual-pol modes deliver better estimations of DoP in urban areas compared to other CP modes. However, this result should be confirmed by studying other polarimetric images.

4. CONCLUSION

Maximum likelihood estimators of the degree of polarization in SAR compact polarimetry were proposed. The performance of these estimators was then evaluated in six CP modes using a RADARSAT-2 single-look fully polarimetric image of San Francisco Bay, CA. The data set consists of three main regions: ocean, urban, and vegetated areas. MLEs of DoP, based on two intensity images, deliver good performance in different CP modes for ocean and vegetated regions. However, in urban areas, DoP is better estimated in traditional dual-pol modes, i.e., (H-H, H-V) and (V-H,V-V). Future work will concentrate on analyzing the performance of these estimators in diverse urban areas.

5. REFERENCES

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