OPTIMAL PARAMETER ESTIMATION IN HETEROGENEOUS CLUTTER FOR HIGH RESOLUTION POLARIMETRIC SAR DATA

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1. INTRODUCTION

The recently launched polarimetric SAR (POLSAR) systems are now capable of producing high quality images of the Earth's surface with meter resolution. The goal of the estimation process is to derive the scene signature from the observed data set. In the case of spatially changing surfaces ("heterogeneous" or "textured" scenes) the first step is to define an appropriate model describing the dependency between the polarimetric signature and the observable as a function of the speckle. In general, the multiplicative model has been employed for POLSAR data processing as a product between the square root of a scalar positive quantity (texture) and the description of an equivalent homogeneous surface (speckle) [1]. The objective of this paper is to present a new parameter estimation technique based on the consistent Spherically Invariant Random Vectors (SIRV) model.

2. SIRV CLUTTER MODEL WITH NORMALIZED TEXTURE

The SIRV is a class of non-homogeneous Gaussian processes with random variance known also as Gaussian scale mixture or compound Gaussian model. The complex m-dimensional measurement \mathbf{k} is defined as the product between the independent complex circular Gaussian vector $\zeta \sim \mathcal{N}\left(0, [T]\right)$ (speckle) with zero mean and covariance matrix $[T] = E\{\zeta\zeta^{\dagger}\}$ and the square

root of the positive random variable ξ (representing the texture):

$$\mathbf{k} = \sqrt{\xi} \cdot \zeta. \tag{1}$$

It is important to notice that in the SIRV definition, the PDF of the texture random variable is not explicitly specified. As a consequence, SIRVs describe a whole class of stochastic processes.

For POLSAR clutter, the SIRV product model is the product of two separate random processes operating across two different statistical axes [2]. The polarimetric diversity is modeled by the multidimensional Gaussian kernel. The randomness of spatial variations in the radar backscattering from cell to cell is characterized by ξ . Relatively to the polarimetric axis, the texture random variable ξ can be viewed as a unknown deterministic parameter from cell to cell.

The texture and the covariance matrix unknown parameters can be estimated from ML theory. For N i.i.d. secondary, let $L_{\mathbf{k}}(\mathbf{k}_1,...,\mathbf{k}_N|[T],\xi_1,...,\xi_N)$ be the likelihood function to maximize with respect to [T] and ξ_i :

$$L_{\mathbf{k}}(\mathbf{k}_{1},...,\mathbf{k}_{N}|[T],\xi_{1},...,\xi_{N}) = \frac{1}{\pi^{mN}\det\{[T]\}^{N}} \times \prod_{i=1}^{N} \frac{1}{\xi_{i}^{m}} \exp\left(-\frac{\mathbf{k}_{i}^{\dagger}[T]^{-1}\mathbf{k}_{i}}{\xi_{i}}\right). \tag{2}$$

The corresponding ML estimators are given by [3]:

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_{1}, \dots, \mathbf{k}_{N} | [T], \xi_{1}, \dots, \xi_{N})}{\partial \xi_{i}} = 0 \Leftrightarrow \widehat{\xi}_{i} = \frac{\mathbf{k}_{i}^{\dagger} [T]^{-1} \mathbf{k}_{i}}{m}, \tag{3}$$

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_{1}, \dots, \mathbf{k}_{N} | [T], \xi_{1}, \dots, \xi_{N})}{\partial [T]} = 0 \Leftrightarrow [\widehat{T}] = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_{i} \mathbf{k}_{i}^{\dagger}}{\xi_{i}}.$$
 (4)

As the variables ξ_i are unknown, the following normalization constraint on the texture parameters insures that the ML estimator of the speckle covariance matrix is the Sample Covariance Matrix (SCM):

$$[\widehat{T}] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^{\dagger} = [\widehat{T}]_{SCM} \Leftrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \mathbf{k}_i^{\dagger} \left(1 - \frac{1}{\xi_i} \right) = [0_m].$$
 (5)

The generalized ML estimator for ξ_i are obtained by introducing Eq. 5 in Eq. 3.

3. SIRV CLUTTER MODEL WITH NORMALIZED COVARIANCE MATRIX

Let now the covariance matrix be of the form $[T] = \sigma[M]$, such that $Tr\{[M]\} = 1$. The product model form Eq. 1 can be also written as:

$$\mathbf{k} = \sqrt{\tau} \cdot \mathbf{z},\tag{6}$$

where $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, [M])$. σ and ξ are two scalar positive quantities such that $\tau = \sigma \cdot \xi$.

Using the same procedure as in Sect. 2, the corresponding texture and normalized covariance ML estimators are given by:

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_{1}, \dots, \mathbf{k}_{N}; [M], \tau_{1}, \dots, \tau_{N})}{\partial \tau_{i}} = 0 \Leftrightarrow \widehat{\tau}_{i} = \frac{\mathbf{k}_{i}^{\dagger} [M]^{-1} \mathbf{k}_{i}}{m}, \tag{7}$$

$$\frac{\partial \ln L_{\mathbf{k}}(\mathbf{k}_{1}, ..., \mathbf{k}_{N}; [M], \tau_{1}, ..., \tau_{N})}{\partial [M]} = 0 \Leftrightarrow [\widehat{M}] = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_{i} \mathbf{k}_{i}^{\dagger}}{\tau_{i}}.$$
 (8)

Given the fact that the covariance matrix is normalized, it is possible to compute the generalized ML estimator of [M] as

the solution of the following recursive equation:

$$[\widehat{M}]_{FP} = f([\widehat{M}]_{FP}) = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \mathbf{k}_i^{\dagger}}{\mathbf{k}_i^{\dagger} [\widehat{M}]_{FP}^{-1} \mathbf{k}_i} = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathbf{z}_i \mathbf{z}_i^{\dagger}}{\mathbf{z}_i^{\dagger} [\widehat{M}]_{FP}^{-1} \mathbf{z}_i}.$$
 (9)

This approach has been used in [4] by Conte et al. to derive a recursive algorithm for estimating the matrix [M]. This algorithm consists in computing the Fixed Point of f using the sequence $([M]_i)_{i>0}$ defined by:

$$[M]_{i+1} = f([M]_i). (10)$$

This study has been completed by the work of Pascal et al. [5], [6], which recently established the existence and the uniqueness, up to a scalar factor, of the Fixed Point estimator of the normalized covariance matrix, as well as the convergence of the recursive algorithm whatever the initialization. The algorithm can therefore be initialized with the identity matrix $[\widehat{M}]_0 = [I_m]$.

The generalized ML estimator for τ_i are obtained by introducing $[\widehat{M}]_{FP}$ in Eq. 7.

4. MAIN RESULT

The span (total power) σ can be derived using the covariance matrix estimators presented in Sect. 2 and Sect. 3 as:

$$\widehat{\sigma} = \frac{\mathbf{k}^{\dagger} [\widehat{M}]_{FP}^{-1} \mathbf{k}}{\mathbf{k}^{\dagger} [\widehat{T}]_{SCM}^{-1} \mathbf{k}}.$$
(11)

Note that Eq. 11 is valid when considering N identically distributed linearly independent secondary data with deterministic unknown texture from cell to cell. It can be seen as a double polarimetric whitening filter issued from two equivalent SIRV clutter models: with normalized texture variables and with normalized covariance matrix parameter.

The main advantage of the proposed estimation scheme is that it can be directly applied with standard boxcar neighborhoods. Fig. 1 illustrates the span σ estimation with high resolution POLSAR X-band data acquired by the ONERA RAMSES system with a spatial resolution of approximately 1.5 m. 5×5 boxcar neighborhood has been selected for illustration. The proposed estimator from Fig. 1-(c) exhibits better performances in terms of spatial resolution preservation than the standard span estimator illustrated in Fig. 1-(b).

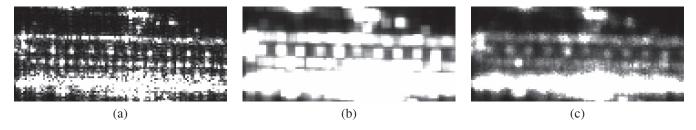


Fig. 1. Brétigny, RAMSES POLSAR data, X-band. (a) initial 1-look span estimated as $\sigma_{SLC} = \mathbf{k}^{\dagger}\mathbf{k}$, (b) 25-look span estimated as $\sigma_{SCM} = \text{Tr}\left\{ [\widehat{T}]_{SCM} \right\}$, and (c) span estimated using $\widehat{\sigma}$ from Eq. 11

Finally, Fig. 2 presents the three SIRV parameters which completely describe the POLSAR data set: the total power, the normalized texture and the normalized covariance matrix.

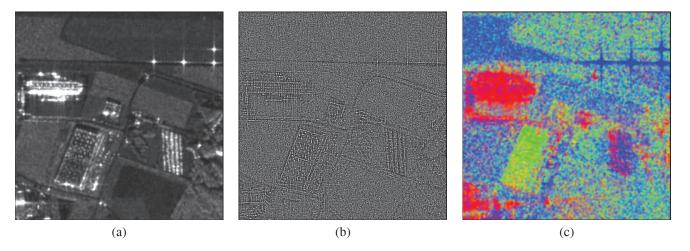


Fig. 2. Brétigny, RAMSES POLSAR data, X-band. (a) span estimated using $\hat{\sigma}$ from Eq. 11, (b) normalized texture ξ , and (c) color composition of the normalized coherency diagonal elements $[M]_{11}$ - $[M]_{33}$ - $[M]_{22}$.

5. CONCLUSIONS

This paper presented a new estimation scheme for optimally deriving clutter parameters with high resolution POLSAR images. The proposed approach couples nonlinear ML estimators with conventional boxcar neighborhoods for taking the local scene heterogeneity into account.

The heterogeneous clutter in POLSAR data was described by the SIRV model. Three estimators were introduced for describing the high resolution POLSAR data set: the span, the normalized texture and the speckle normalized covariance matrix.

In the final version of this paper, authors propose to study the asymptotic distribution of the new span estimator. The estimation bias on homogeneous regions will be assessed also. However, preliminary Monte Carlo studies indicate that the proposed estimator is unbiased at least assimptotically.

6. REFERENCES

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