TITLE: HIGH-RESOLUTION 3-D RADAR IMAGING USING PSEUDO-RANDOM NOISE CODED WAVEFORM

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1. ABSTRACT:

In this paper, high-resolution 3-D radar imaging system configuration using a 3-step image formation procedure is analyzed and discussed [1,2]. To demonstrate the high-resolution of the imaging system, a 3-D facet-model of object is used and computer simulation was conducted.

In the 3-D radar imaging system, an object is illuminated by coherent electromagnetic waves, and the reflected signal is heterodyned with a reference signal. The amplitude and phase information of the reflected signal are recorded for reconstructing an image of the object.

Assume an object is located in the region z > 0 in the fixed radar coordinate system (x, y, z) and the radar is located at (x_T, y_T, z_T =0). There are no multiple reflections among the scatterers of the object. When the radar transmits a signal with a carrier frequency of *f* expressed by $s(t) = \exp(j2\pi ft)$, assuming the largest dimension of the object and the length of the receiving aperture are significantly smaller than the distance between the radar and the object, the distribution of the electromagnetic field on the receiving plane array (*x*-*y*) can be expressed as

$$\begin{aligned} U(x,y,t) = \iiint_{object} O(x_n,y_n,z_n)\frac{f}{jz_n^2c}\exp\{-j2\pi f(\frac{2z_n}{c})\}\cdot \\ \exp\{-j2\pi f\left[\frac{(x-x_n)^2+(y-y_n)^2+(x_T-x_n)^2+(y_T-y_n)^2}{2z_nc}\right]\}\cdot \\ \exp(j2\pi ft)\,dx_n dy_n dz_n, \end{aligned} \quad (1)$$

where *c* is the speed of wave propagation, and $O(x_n,y_n,z_n)$ is the object distribution function described in terms of the reflectivity. Then, the received signal at each receiver aperture is multiplied by the quadrature reference signals and low-pass filtered. The base-band signal becomes

$$\begin{aligned} s_H(x,y) = \iiint_{object} O(x_n,y_n,z_n)\frac{f}{jz_n^2c}\exp\{-j2\pi f(\frac{2z_n}{c})\}\cdot \\ \exp\{-j2\pi f\left[\frac{(x-x_n)^2+(y-y_n)^2+x_n{}^2+y_n{}^2}{2z_nc}\right]\}dx_n dy_n dz_n, \end{aligned} \quad (2)$$

where the radar is assumed at (x_T=0,y_T=0,z_T=0). The object distribution function $O(x_n,y_n,z_n)$ can be represented slice by slice along the z_n axis as follows:

$$O(x_n,y_n,z_n) = \sum_{k=1}^{K} O_k(x_n,y_n)\delta(z_n - z_k),$$

where $O(x_n,y_n)$ is the reflection coefficient of the object at the point (x_n,y_n) in the *k*th slice. Thus, the heterodyned signal can be written as follows:

$$s_H(x,y)=\sum_{k=1}^{K}\exp\{-j2\pi f(\frac{2z_k}{c}+\frac{x^2+y^2}{2z_k c})\}$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{O_k(x_n,y_n)\frac{f}{jz_k^2 c}\cdot$$
$$\exp\{-j2\pi f\left[\frac{x_n{}^2+y_n{}^2}{2z_k c}\right]\} \tag{3}$$
$$\exp\{-j2\pi f\left[\frac{xx_n+yy_n}{z_k c}\right]\}dx_n dy_n.$$

From the Eq. (3), we can clearly see that three operations on the heterodyned signal $s_H(x,y)$ are needed to reconstruct a 3-D image of the object [1,2]:

(1) Focus operation - multiply $s_H(x,y)$ with a focus factor $\exp\{-j2\pi f(\frac{2z_n}{c}+\frac{x^2+y^2}{2z_n c})\}$.

After the focus operation, the Eq (3) becomes

$$s_H(x,y)\cdot\exp\{-j2\pi f(\frac{2z_n}{c}+\frac{x^2+y^2}{2z_n c})\}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{O_k(x_n,y_n)\frac{f}{jz_n^2 c}\cdot$$
$$\exp\{-j2\pi f\left[\frac{x_n{}^2+y_n{}^2}{2z_n c}\right]\} \tag{4}$$
$$\exp\{-j2\pi f\left[\frac{xx_n+yy_n}{z_n c}\right]\}dx_n dy_n+I(x,y,z_n,z_k)$$

where

$$I(x,y,z_n,z_k)=\sum_{\substack{k=1\\k\neq n}}^{K}\exp\{-j2\pi f[\frac{2(z_n-z_k)/c+(x^2+y^2)}{2c(1/z_n-1/z_k)}]\}$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{O_k(x_n,y_n)\frac{f}{jz_k^2 c}\cdot$$
$$\exp\{-j2\pi f\left[\frac{x_n{}^2+y_n{}^2}{2z_k c}\right]\}$$
$$\exp\{-j2\pi f\left[\frac{xx_n+yy_n}{z_k c}\right]\}dx_n dy_n.$$

(2) Fourier transform - applying the Fourier transform on both sides of (4) using a kernel function $\exp\{-j2\pi f(\frac{xx_n+yy_n}{z_n c})\}$, thus, we have

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s_H(x,y)\cdot\exp\{-j2\pi f(\frac{2z_n}{c}+\frac{x^2+y^2}{2z_n c})\}\cdot\exp\{-j2\pi f(\frac{xx_n+yy_n}{z_n c})\}dxdy$$
$$=O_n(x_n,y_n)\cdot\frac{1}{z_n}\exp\{-j2\pi f\left[\frac{x_n^2+y_n^2}{z_n c}\right]\}dx_n dy_n \tag{5}$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}I(x,y,z_n,z_k)\cdot\exp\{-j2\pi f(\frac{xx_n+yy_n}{z_n c})\}dxdy.$$

(3) Post-multiplier operation:
After multiplying by a factor of

$$z_n \exp\{j2\pi f(\frac{x_n^2 + y_n^2}{z_n c})\},$$

the constructed image slice at z_n can be obtained:

$$\begin{aligned} O_n(x_n, y_n) &= z_n \exp\{j2\pi f(\frac{x_n^2 + y_n^2}{z_n c})\} \cdot \\ &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s_H(x, y) \cdot \exp\{-j2\pi f(\frac{2z_n}{c} + \frac{x^2 + y^2}{2z_n c})\} \cdot \exp\{-j2\pi f(\frac{xx_n + yy_n}{z_n c})\}dxdy \\ &+ z_n \exp\{j2\pi f(\frac{x_n^2 + y_n^2}{z_n c})\} \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y, z_n, z_k) \cdot \exp\{-j2\pi f(\frac{xx_n + yy_n}{z_n c})\}dxdy. \end{aligned} \quad (6)$$

where the second term is a cross interference

$$CI(x_n, y_n, z_n, z_k) = z_n \exp\{j2\pi f(\frac{x_n^2 + y_n^2}{z_n c})\} \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y, z_n, z_k) \cdot \exp\{-j2\pi f(\frac{xx_n + yy_n}{z_n c})\}dxdy.$$

The cross interference represents the effect of the scatterers at range z_k on the constructed image slice at range z_n. The magnitude of the cross interference depends only on the range resolution of the image construction process. If the range resolution is very high, then there will be no cross interference. To achieve high resolution in the (x-y) plane, the effective size of the receiving array must be increased. In practice, the synthetic aperture method may be used by scanning a lined sensor array or even a single sensor to synthesize a large aperture. A useful approach to increasing the range resolution is to use a modulated signal with wide-bandwidth and pulse compression technique. Pseudo-random noise (PN) coded waveform have been used effectively in radar, sonar and deep-space missions [3-6]. In PN code systems, by using the correlation processing, the range resolution can be resolved to within one code-bit duration or even in a fraction of one code-bit duration. PN code system can also provide noise immunity.

The transmitted PN coded waveform can be expressed as

$$s_T(t) = P_{PN}(t) \cdot \exp\{j2\pi ft\},$$

where $P_{PN}(t)$ is a two-level PN code. For the type of phase shift key modulation, when the digit in the PN sequence is 1, the level of $P_{PN}(t)$ is -1; the digit is 0, the level is +1. Suppose the input signal to the demodulation is

$$s_r(t) = A \cdot P_{PN}(t - \tau) \cdot \exp\{j2\pi f(t - \tau)\}.$$

Then, after mixing with the reference of the transmitted carrier signal, the output of the coherent demodulation becomes

$$s_O(t) = (A/2) \cdot P_{PN}(t - \tau) \cdot \exp\{j2\pi f\tau\}.$$

After a low-pass filtering, the output of the demodulated signal is correlated with a delayed replica of the transmitted PN code. The resulting output of the correlation is

$$\begin{aligned} s_C(\tau, \tau_0) &= (A/2) \cdot [\int_0^T P_{PN}(t - \tau) \cdot P_{PN}(t - \tau_0) dt] \cdot \exp\{j2\pi f\tau\} \\ &= (A/2) R_{PN}(\tau - \tau_0) \cdot \exp\{j2\pi f\tau\}. \end{aligned}$$

For $\tau_0 = \tau$, $s_C(\tau, \tau_0)$ reaches its maximal value, and for $\tau_0 \neq \tau$, the value of $s_C(\tau, \tau_0)$ is very low. Thus, high range resolution can be achieved.

For an ideal situation, when the autocorrelation function $R_{PN}(\tau)$ is a delta function $\delta(\tau)$, there will be no cross interference term and the image can be constructed with very high range resolution.

A facet-model of 3-D object is used for computer simulation study. The simulation result shows the reconstructed high-resolution 3-D radar image of the object using a PN coded waveform. Further study of synthesizing 2-D receiving aperture array with synthetic aperture method is proposed.

2. REFERENCES

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3. BIOGRAPH

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