Optimal sensor positioning for ISAR imaging

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1 Problem statement

One of the major problems in ISAR imaging is that the image formed at the end of the process is a 2D projection of the true target reflectivity onto the ISAR Image Projection Plane (IPP). The orientation of this plane depends on the sensor position relative to the target and on the target motion, which, in the case of non-cooperative targets, is not under the radar operator's control. The result of this is that the target projection seen in the ISAR image becomes arbitrary, which makes the interpretation of the ISAR image, and consequently the recognition of the target, a very difficult task. In addition to this problem, the ISAR image cross-range resolution is unknown a priori because it also depends on the target's own motion. To circumvent these issues, it is often assumed that data has to be collected over a long duration to obtain at least one suitable frame with the desirable resolution and IPP. It is assumed that there exists an imaging interval where the combination of the target's own motion and the sensor position would produce the desired result, e.g. a top view or a side view ISAR image with sufficient cross-range resolution which is useful for classification. Unsurprisingly, this approach does not leave any degrees of freedom to try and force a more desirable result. If, however, we open the problem to that of choosing the position of the ISAR sensor, it might be argued that there exists an optimal radar position that provides more desirable result. In this paper, without introducing any a priori knowledge, apart from physical constraints,a mathematical framework for estimating the optimum sensor position for imaging objects with 3D rotational motion is discussed. Specifically, simple tools will be provided that will solve the problem of positioning an ISAR sensor in order to maximise the probability of obtaining an ISAR image with a desired IPP by constraining the achievable cross-range resolution. It is worth mentioning that the solution of this problem also finds a natural application in the problem of positioning a given number of sensors in a multi-static configuration, although this extension will not be treated directly in this paper.

2 Signal model

Inverse Synthetic Aperture Radar (ISAR) is often interpreted as the result of a motion-induced Doppler formation, which sometimes simplifies the understanding of the cross-range image formation. This approach will be followed to derive some mathematical expressions that will lead to the problem solution [1, 2]. After radial motion compensation, the noiseless component of the radar received signal can be written as a volumetric integral function of the target reflectivity by taking into account both the radar parameters and the target-radar geometry [3].

$$S_R\left(f,t\right) = \int_V \xi(\mathbf{x}) \exp\left\{-j\frac{4\pi f}{c}\left[\mathbf{x}^T \cdot \mathbf{i}_{LoS}(t)\right]\right\} d\mathbf{x}$$

where f and t represent the frequency and time domain, $\mathbf{x} = [x_1, x_2, x_3]^T$ is the coordinate vector of an arbitrary point on the target with respect to a cartesian coordinate system embedded in the target, c is the speed of light and $\mathbf{i}_{LOS} = [\cos\theta_a \cos\theta_e\, , \sin\theta_a \cos\theta_e\, , \sin\theta_e]^T$ represents the target-to-radar Line of Sight (LoS) as a function of azimuth θ_a and elevation θ_e. Once the translational motion component is removed from the received signal, the target can be seen as animated by rotational-only motions.

Therefore, at an arbitrary time t, such motions can be represented by means of a rotation vector velocity $\mathbf{\Omega}(\mathbf{t}) = [\Omega_1(t)\ , \Omega_2(t)\ , \Omega_3(t)]^T$, which can be interpreted as roll, pitch and yaw rate components. The instantaneous Doppler frequency generated by the target rotation is first derived in order to relate it to 1) the target motions, 2) the scatterer's position and 3) the radar's position. Moreover, in order to separate the three contributions, a matrix notation is introduced, as follows:

$$f_d(t) = \frac{2}{\lambda}\left[\mathbf{\Omega}_{eff}(t) \times \mathbf{x}\right] = \frac{2}{\lambda}\left[\mathbf{\Omega}^T(t)\,\mathbf{Lx}\right]$$

where the symbol $\times$ indicates a cross-product, $\mathbf{\Omega}_{eff}(t)$ is the *effective rotation vector*, which can be expressed as $\mathbf{\Omega}_{eff}(t) = \mathbf{i}_{LOS} \times (\mathbf{\Omega}(\mathbf{t}) \times \mathbf{i}_{LOS})$, and where $\mathbf{L}$ is a 3x3 matrix, with the following elements:

$$\begin{array}{c} L_{11} = L_{22} = L_{33} = 0 \\ L_{12} = -L_{21} = \sin\theta_e \\ L_{31} = -L_{13} = \cos\theta_e \sin\theta_a \\ L_{23} = -L_{32} = \cos\theta_a \cos\theta_e \end{array} \tag{1}$$

It is worth noting that the first equality in (1) satisfies the physical constraint that a rotation with respect to an axis does not produce any motion along the same axis, i.e. no Doppler is produced along the same axis. It should also be noted that the three ingredients, namely the target motions, the scatterer's position and the radar's position are separated in three factors: $\mathbf{\Omega}$, $\mathbf{x}$ and $\mathbf{L}$.

The instantaneous Doppler frequency can be also written as the sum of three coordinate-related components

$$f_d(t) = L_1(t)\,x_1 + L_2(t)\,x_2 + L_3(t)\,x_3$$

where

$$\begin{array}{c} L_1(t) = \Omega_2(t)\,L_{21} + \Omega_3(t)\,L_{31} \\ L_2(t) = \Omega_1(t)\,L_{12} + \Omega_3(t)\,L_{32} \\ L_3(t) = \Omega_1(t)\,L_{13} + \Omega_2(t)\,L_{23} \end{array} \tag{2}$$

$L_1(t)$, $L_2(t)$ and $L_3(t)$ are here defined as the *Doppler generating factors* for the three spatial components. In other words, a scatterer will produce a Doppler component proportional to the product of its coordinate and the relative Doppler generating factor. As an example, a scatterer located at $(x_1, 0, 0)$ will produce a Doppler component proportional to $L_1(t)$ and its coordinate x_1. Also, note that it will produce a non-zero Doppler component only if $L_1(t) \neq 0$. The terms in (2) can be forced to zero in pairs by selecting the suitable azimuth and elevation angles, which define the position of the radar, in order to allow only one of the target's dimension to produce a Doppler component. This provides the means for forming desired target projections, such as front, side and top views.

3 Image Projection Plane forcing constraint

Pure front, side and top views can be obtained by forcing the relative Doppler generating factors to zero and by constraining the values of the azimuth and elevation angle to zero, according to the desired view. To limit the amount of pages in this abstract, only the pure side view will be mathematically derived, whereas a complete mathematical tractation of all the views will be included in the final paper. In addition, a mixed front/side view will be considered where the target's height information is preserved in the formed ISAR image.

3.1 Pure side view

A side view can be obtained by forcing the terms $L_2(t)$ to zero and by constraining $\theta_a = 0$. Therefore,

$$L_2(t) = \Omega_1(t)\sin\theta_e - \Omega_3(t)\cos\theta_a\cos\theta_e = 0,\ subject\, to\, \theta_a = 0 \tag{3}$$

The solution of the non-linear equation in (3) can be written as $\theta_e(t) = \arctan\left[\frac{\Omega_3(t)}{\Omega_1(t)}\right]$

3.2 Mixed front/side view

A mixed front/side view, with no top view component, can be obtained by forcing both terms $L_1(t)$ and $L_2(t)$ to zero. Four symmetric solutions exist that can be reduced to a single solution in the domain $(\theta_a, \theta_e) \in \left\{\left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]\right\}$. The solution can be written in a closed form, as follows:

$$\begin{cases} \theta_a(t) = \arctan\left[\frac{\Omega_2(t)}{\Omega_1(t)}\right] \\ \theta_e(t) = \arctan\left[\frac{\Omega_3(t)}{\sqrt{\Omega_1(t)+\Omega_2(t)}}\right] \end{cases}$$

It is worth pointing out that the Doppler component is driven only by the position of the scatterer along the height. Therefore, information about the target's height can be inferred from the Doppler spread of the target's ISAR image.

4 Cross-range Resolution constraint

The cross-range resolution can be calculated easily when the effective rotation vector can be assumed sufficiently constant during the Coherent Processing Interval (CPI), as follows

$$\delta_{cr} = \frac{c}{2 f_0 \Omega_{eff} T_{ob}} \tag{4}$$

where c is the speed of light, f_0 is the carrier frequency, T_{ob} is the CPI and Ω_{eff} is the modulus of $\mathbf{\Omega}_{eff}$.

As detailed in (4), the cross-range resolution depends on several parameters. Since the only degree of freedom available is the sensor position, in order to minimise the cross-range resolution, Ω_{eff} must be maximised. It can be simply demonstrated that Ω_{eff} is maximum when the LoS is orthogonal to the target's rotation vector $\mathbf{\Omega}$. Such a condition can be applied by forcing the inner product between LoS and $\mathbf{\Omega}$ to be zero, i.e. $\mathbf{\Omega} \cdot \mathbf{i_{LoS}} = \cos\theta_a \cos\theta_e \Omega_1 + \sin\theta_a \cos\theta_e \Omega_2 + \sin\theta_e \Omega_3 = 0$, which admits an infinite number of solutions, as follows:

$$\theta_e = -\arctan\left[\frac{\Omega_1 \cos\theta_a + \Omega_2 \sin\theta_a}{\Omega_3}\right] \tag{5}$$

In general, the solution for the problem of minimising the cross-range resolution does not represent a solution for the problem of obtaining a desired IPP. Therefore, we can consider the problem of obtaining a given IPP without sacrificing too much resolution.

4.1 Cross-range resolution constrained IPP selection

For a given couple of (θ_a, θ_e) that provides a desired IPP, a relative cross-range resolution is obtained, which can be calculated by using (4). A desired IPP will be considered effective if the resolution obtained does not increase of a factor greater than γ, in formula

$$\delta_{cr} < \gamma \delta_{cr}^{(min)} \tag{6}$$

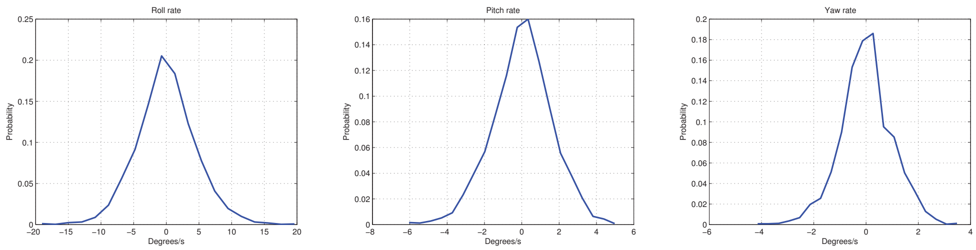

Figure 1: Histograms of measured roll, pitch and yaw rates

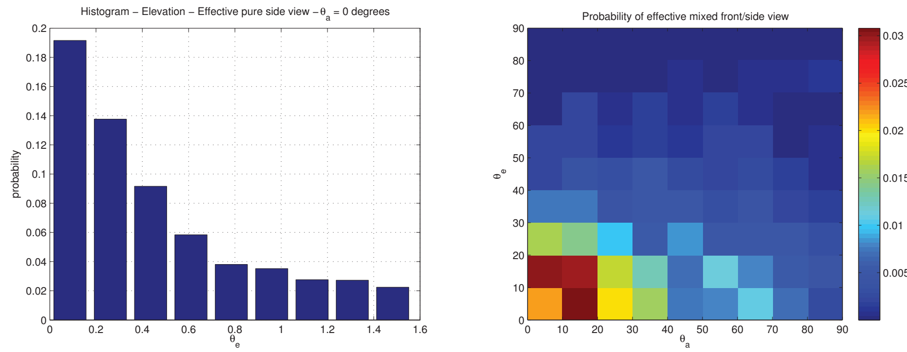

Figure 2: Side and mixed front/side view sensor position histograms

where δ_{cr}^{min} is obtained by positioning the sensor in any pair $\left(\theta_a, -\arctan\left[\frac{\Omega_1 \cos\theta_a + \Omega_2 \sin\theta_a}{\Omega_3}\right]\right)$, as suggested by the solution in (5).

5 Preliminary results

Measured roll, pitch and yaw motion data of small sea vessel were obtained by instrumenting an Inertial Measurement Unit (IMU) on-borad the vessel are used to provide preliminary results. Roughly 3500 samples at a sample rate of 0.2 seconds were acquired. A resolution constraint equal to $\gamma = 3$ was applied. The histograms of the measured roll, pitch and yaw rates are shown in Fig. 1.

The results displayed in Fig. 2 show that the probability of obtaining a side view ISAR image is more concentrated around values of the elevation angles close to zero. A mixed front/side view is likely to be obtained if the sensor is positioned at close to zero azimuth and zero elevation, although, the maximum probability density does not occur at values of the angles equal to zero.

More results will be shown in the final paper.

References

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