# VEGETATION ISOLINE EQUATIONS FOR ANALYSIS OF HYPER-SPECTRAL DATA WITH HIGHER ORDER INTERACTION TERMS

Mumenori Miura, Kenta Obata, and Hiroki Yoshioka

Aichi Prefectural University
Department of Applied Information Science and Technogoly
1522-3 Kumabari, Nagakute, Aichi 480-1198, Japan
yoshioka@ist.aichi-pu.ac.jp

#### 1. BACKGROUND

Parameter retrieval from remotely sensed satellite data often involves algebraic manipulation of band reflectance focusing on several wavelengths. One example is regression analysis of biophysical parameters (objective variable), which uses parameters obtained from band rationing as independent variable. The logic behind the analysis relies heavily on the existence of certain relationship between the objective variable and the variable obtained from band manipulations [1].

The equation which describes such a relationship is basically equivalent to a relationship among reflectances from different wavelengths. Therefore, derivation of such relationships is a key to better understanding of parameter retrievals from spectral data [2, 3, 4, 5]. This paper tries to contribute the development of those band-to-band relationship of spectral reflectance for the use of parameter retrieval.

In our previous work we derived the equations of vegetation isoline by taking into account the higher order interaction terms under the following two limitations [6]. The first one is about the wavelength; we derived the equation only for the relationships between red and NIR bands. In order to apply the derived relationships into wide range of wavelength, this restriction needs to be relaxed. The second limitation is about the assumption of canopy model; the derivation was carried out by assuming that vegetation canopy covers the entire surface. Due to this assumption, the derived expression is not applicable to the case of partially covered canopy. This paper explains our approach to overcome these limitations.

#### 2. OBJECTIVES

This paper is to explain our recent progress about derivations of a series of vegetation isoline equations. Our focus is to investigate accuracies of derived isoline equations in the context of hyper-spectral data analysis. The objectives of this paper are three folds;

- 1. Derive vegetation isoline equations between arbitrary two bands with higher order interaction terms under partially vegetated target (fraction of vegetation cover is less than unity),
- 2. Evaluate its accuracy with radiative transfer model of vegetation canopy, and
- 3. Compare the accuracy with the other versions of vegetation isoline equations with fewer number of interaction terms.

# 3. THREE FORMS OF VEGETATION ISOLINE EQUATIONS

There are three forms of vegetation isoline equations, difference of which is the degree of approximation. The accuracy depends on the number of interaction term included in the original formulations of reflectance model. The three forms of isoline are summarized below, and the detailed explanations will be provided in the presentation.

## 3.1. Dual first order approximated vegetation isoline equation (DF-VIE)

The vegetation isoline equation is a form of relationship between two reflectances ( $\rho_{\lambda_1}$  and  $\rho_{\lambda_2}$ ) observed at different wavelengths ( $\lambda_1$  and  $\lambda_2$ ). The linear form of vegetation isoline equation was intorduced by Yoshioka et al. [1, 2]. The equation was

derived by truncating second-order interaction terms for reflectance of two bands (which is usually red and NIR reflectance). The derived isoline equation (dual first order approximated vegetation isoline equation, DF-VIE) is as follows.

$$\rho_{\lambda_2} = \alpha_1 \rho_{\lambda_1} + \beta_1 + \epsilon_1 \tag{1}$$

$$\alpha_1 = m \frac{\omega T_{\lambda_2}^2 + 1 - \omega}{\omega T_{\lambda_1}^2 + 1 - \omega} \tag{2}$$

$$\beta_1 = n(\omega T_{\lambda_2}^2 + 1 - \omega) - \alpha_1 \omega R_{u\lambda_1} + \omega R_{u\lambda_2} \tag{3}$$

$$\epsilon_1 = \omega O(R_{s\lambda_2}^2) - \alpha_1 \omega O(R_{s\lambda_1}^2) \tag{4}$$

$$\rho_{\lambda_2} \approx \alpha_1 \rho_{\lambda_1} + \beta_1 \tag{5}$$

The form of DF-VIE is simply a linear form, which makes the use of this expression easier owing to this simplicity. The drawback of this simplification is accuracy. The accuracy can be improved by retaining the second order interaction terms.

# 3.2. Dual second order approximated vegetation isoline equation (DS-VIE)

This form of isoline equation, newly derived in this study, can be obtined by retaining up to the second order interaction term for both wavelengths (dual second order approximated vegetation isoline equation, DS-VIE). The final form becomes as follows.

$$\rho_{\lambda_2} = \alpha_2 \rho_{\lambda_1} + \beta_2 \sqrt{\alpha_2' \rho_{\lambda_1} + \delta_2 - \epsilon_2'} + \gamma_2 + \epsilon_2 \tag{6}$$

$$\alpha_2 = \frac{m^2 T_{\lambda_2}^2 R_{d\lambda_2}}{T_{\lambda_1}^2 R_{d\lambda_1}} \tag{7}$$

$$\alpha_2' = 4\omega T_{\lambda_1}^2 R_{d\lambda_1} \tag{8}$$

$$\beta_2 = m \left\{ \frac{2n\omega T_{\lambda_2}^2 R_{d\lambda_2} + \omega T_{\lambda_2}^2 + 1 - \omega}{2\omega T_{\lambda_1}^2 R_{d\lambda_1}} - \frac{m T_{\lambda_2}^2 R_{d\lambda_2} (\omega T_{\lambda_1}^2 + 1 - \omega)}{2\omega T_{\lambda_1}^4 R_{d\lambda_1}^2} \right\}$$
(9)

$$\gamma_2 = n^2 \omega T_{\lambda_2}^2 R_{d\lambda_2} + n \left( \omega T_{\lambda_2}^2 + 1 - \omega \right) + \omega R_{u\lambda_2} - \omega \alpha_2 R_{u\lambda_1} - \beta_2 \left( \omega T_{\lambda_1}^2 + 1 - \omega \right) \tag{10}$$

$$\delta_2 = (\omega T_{\lambda_1}^2 + 1 - \omega)^2 - 4\omega^2 T_{\lambda_1}^2 R_{d\lambda_1} R_{u\lambda_1}$$
(11)

$$\epsilon_2 = \omega O(R_{s\lambda_2}^3) - \omega \alpha_2 O(R_{s\lambda_1}^3) \tag{12}$$

$$\epsilon_2' = 4\omega^2 T_{\lambda_1}^2 R_{d\lambda_1} O(R_{s\lambda_1}^3) \tag{13}$$

$$\rho_{\lambda_2} \approx \alpha_2 \rho_{\lambda_1} + \beta_2 \sqrt{\alpha_2' \rho_{\lambda_1} + \delta_2} + \gamma_2 \tag{14}$$

Although the above form of isoline equation (DS-VIE) is indeed much better in accuracy than DF-VIE, the form becomes rather complicated which makes the use of the equations much harder comparing to DF-VIE. A expression of simpler form without loosing the accuracy from the DS-VIE is preferable. Such a form is explained in the next subsection.

## 3.3. Semi-second order approximated vegetation isoline equation (SS-VIE)

The derivation of simpler form than DS-VIE without loosing accuracy was derived by retaining the second order interaction term only for one wavelength (semi-second order approximated vegetation isoline equation, SS-VIE). The results of derivation are summarized as follow.

$$\rho_{\lambda_2} = \alpha_3 \rho_{\lambda_1}^2 + \beta_3 \rho_{\lambda_1} + \gamma_3 + \epsilon_3 \tag{15}$$

$$\alpha_3 = m^2 \frac{\omega T_{\lambda_2}^2 R_{d\lambda_2}}{(\omega T_{\lambda_1}^2 + 1 - \omega)^2} \tag{16}$$

$$\beta_3 = x - 2\alpha_3 \omega R_{u\lambda_1} \tag{17}$$

$$\gamma_3 = \alpha_3 \omega^2 R_{u\lambda_1}^2 - x\omega R_{u\lambda_1} + y + \omega R_{u\lambda_2} \tag{18}$$

$$\epsilon_3 = \alpha_3 \omega \left\{ O(R_{s\lambda_1}^2) \right\}^2 + \left( 2\alpha_3 \omega^2 R_{u\lambda_1} - \omega x - 2\alpha_3 \omega \rho_R \right) O(R_{s\lambda_1}^2) + \omega O(R_{s\lambda_2}^3) \tag{19}$$

$$x = 2mn \frac{\omega T_{\lambda_2}^2 R_{d\lambda_2}}{\omega T_{\lambda_1}^2 + 1 - \omega} + m \frac{\omega T_{\lambda_2}^2 + 1 - \omega}{\omega T_{\lambda_1}^2 + 1 - \omega}$$
(20)

$$y = n^2 \omega T_{\lambda_2}^2 R_{d\lambda_2} + n(\omega T_{\lambda_2}^2 + 1 - \omega)$$

$$\tag{21}$$

$$\rho_{\lambda_2} \approx \alpha_3 \rho_{\lambda_1}^2 + \beta_3 \rho_{\lambda_1} + \gamma_3 \tag{22}$$

Since the term of squire root is now vanished in this form, the expression is simpler, and hence, in general, easier to use for data analysis.

#### 4. NUMERICAL SIMULATION

The derived expressions of vegetation isoline will be demonstrated numerically with a couple of leaf and canopy reflectance models. PROSPECT [7, 8] and GeoSAIL [9, 10] were employed to simulate leaf and canopy optical properties (i.e. reflectance and transmittance), respectively. We assumed LAI values from 1.0, 2.0 and 3.0. The soil line slope and offset were set to be 1.0 and 0.0, respectively. The wavelength range we considered is from 400 [nm] to 1700 [nm], covering visible to shortwave infrared. Reflectance from a system of canopy-soil layers were obtained at discrete values of wavelength which are located at every 2.5 [nm] interval within the wavelength range. The simulated reflectance spectra is equivalent to 520 bands of hyperspectral data.

The isoline parameters introduced in the previous section were computed from the simulated reflectance spectra for every combination of two reflectances of 520 bands, resulting that we simulated 520 by 520 of vegetation isolines for each canopy (of different LAI).

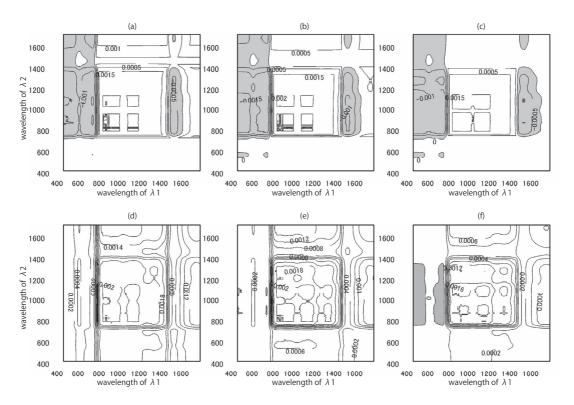
# 5. RESULTS

The error is defined as a difference of reflectance between the one obtained from numerical simulation (which has no truncation error) and the one approximated by the isoline equations of the three forms. The errors are compared among the three isoline equations for all the 520 by 520 combinations of wavelength. Figure 1 summarizes the differences of the error among the three isolines. The upper three figures (Fig.1(a)-(c)) are comparisons between DF-VIE and SS-VIE. The differences in error for all the combinations of wavelength are plotted in Figs.1(a), (b), and (c) for LAI=1.0, 2.0 and 3.0, respectively. Each point of figure corresponds to a pair of reflectance at different wavelength ( $\lambda_1$  and  $\lambda_2$  in x and y axis, respectively). When the error of SS-VIE is smaller than that of DF-VIE, the point was colored. As can be seen from these figures, SS-VIE is better approximation (higher accuracy) than DF-VIE when the combination of the wavelength is visbile and infrared/shortwave. These combinations of wavelength are most frequently used pair of wavelength for analysis of vegetated surface.

Similarly, the errors were compared between DS-VIE and SS-VIE in Figs.1(d)-(f) (lower three figures). From the figures, the difference in error between the two isoline equations are almost comparable despite the fact that SS-VIE is less number of higher order interaction terms than DS-VIE. Although DS-VIE were expected to be more accurate than SS-VIE, SS-VIE is even better approximation than DS-VIE in some cases (indicated by colored region in Fig.1(f).) The reasons of having results and the mechanisms will be fully explained in the presentation.

## 6. REFERENCES

[1] H. Yoshioka, T. Miura, and A. R. Huete, "Analysis of vegetation isolines in red-NIR reflectance space," *Remote Sens. Environ.*, vol. 74, no. 2, pp. 313–326, 2000.



**Fig. 1.** Error differences between DF-VIE and SS-VIE (upper three figures) and DS-VIE and SS-VIE (lower three) at three LAI values of 1.0 (a, d), 2.0 (b, e), and 3.0 (c, f). The shaded area indicates that SS-VIE is better approximation than the others.

- [2] H. Yoshioka, A. R. Huete, and T. Miura, "Derivation of vegetation isoline equation in red-NIR reflectance space," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 2, pp. 838–848, 2000.
- [3] H. Yoshioka, "Vegetation isoline equations for an atomosphere-canopy-soil system," *IEEE Trans. Geosci. Remote Sens.*, vol. 42, no. 1, pp. 166–175, 2004.
- [4] H. Yoshioka, T. Miura, and A. R. Huete, "An isoline-based translation technique of spectral vegetation index using EO-1 hyperion date," *IEEE Trans. Geosci. Remote Sens.*, vol. 41, no. 6, pp. 1363–1372, 2003.
- [5] A. Kallel, S. L. Hegarat-Mascle, C. Ottle, and L. Hubert-Moy, "Determination of vegetation cover fraction by inversion of a four-parameter model based on isoline parametrization," *Remote Sens. Environ.*, vol. 111, no. 4, pp. 553–566, 2007.
- [6] M. Miura and H. Yoshioka, "Vegetation isoline equation with higher order interaction terms to improve accuracy in retrieval of vegetation biophysical parameter," in *IEEE IGARSS'08*, 2008, vol. 3, pp. 1056–1059.
- [7] S. Jacquemoud and F. Baret, "PROSPECT: a model of leaf optical properties spectra," *Remote Sens. Environ.*, vol. 34, pp. 75–91, 1990.
- [8] S. Jacquemoud, S. L. Ustin, J. Verdebout, G. Schmuck, Andreoli, G., and B. Hosgood, "Estimating leaf biochemistry using the prospect leaf optical properties model," *Remote Sens. Environ.*, vol. 56, pp. 194–202, 1996.
- [9] W. Verhoef, "Light scatteriing by leaf layers with applications to reflectance modeling: The SAIL model," *Remote Sens. Environ.*, vol. 16, pp. 125–141, 1984.
- [10] K. F. Huemmrich, "The geosail model:a simple addition to SAIL model to describe discintinuous canopy reflectance," *Remote Sens. Environ.*, vol. 75, pp. 423–431, 2001.