

# POLARIMETRIC SAR ESTIMATION BASED ON NON-LOCAL MEANS

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## 1. INTRODUCTION

During the past few years, the non-local (NL) means [1] has proved its efficiency for image denoising. This approach assumes there exist enough redundant patterns in images to be used for noise reduction. We suggest that the same assumption can be done for polarimetric synthetic aperture radar (PolSAR) images. In its original version, the NL means deal with additive white Gaussian noise. More recently, several extensions have been proposed for non-Gaussian noise [2] and [3]. This abstract applies the methodology proposed in [3] to PolSAR data. The proposed filter seems to deal well with the statistical properties of speckle noise and the multi-dimensional nature of such data. Results are given on L-Band E-SAR data to validate the proposed method.

## 2. NON-LOCAL MEANS

Non-local (NL) approaches have been proposed by Buades *et. al* in [1] to denoise images damaged by additive white Gaussian noise. While local filters lead to biases and resolution loss, NL techniques are known to efficiently reduce noise and preserve structures. Instead of combining neighbor pixels, the NL means average similar pixels. NL means assume there are enough redundant pixels (pixel having identical noise-free value) in the image to reduce the noise significantly. Let  $v_s$  be the observed noisy value at site  $s$  and  $u_s$  its underlying noise-free value, NL means provides the estimate  $\hat{u}_s$  defined by:

$$\hat{u}_s = \frac{\sum_t w(s, t)v_t}{\sum_t w(s, t)} \quad (1)$$

where  $t$  is a pixel index and  $w(s, t)$  is a data-driven weight depending on the similarity between pixels with index  $s$  and  $t$ . For robustness reason, pixel similarity is evaluated by comparing surrounding patches around  $s$  and  $t$  with the use of the Euclidean distance:

$$w(s, t) = \exp\left(-\frac{\sum_k (v_{s,k} - v_{t,k})^2}{h}\right) \quad (2)$$

where  $_{s,k}$  and  $_{t,k}$  denote respectively the  $k$ -th pixels in the patches centered on  $s$  and  $t$ , and  $h$  is a filtering parameter. Equation (1) and Equation (2) are well adapted to estimate noise-free values and to evaluate patch-similarity when the observed image is damaged by additive white Gaussian noise. We describe in the following how this approach can be extended to handle speckle noise in the polarimetric case.

## 3. WEIGHTED MAXIMUM LIKELIHOOD

The weighted average performed by NL means can be seen as a particular case of the weighted maximum likelihood (WML) estimation. ML based filters assume there exists redundant pixels and search the value which maximizes the likelihood over the set of redundant pixel values. Since this set is unknown, we propose to approach its indicator function with weights, which leads to the WML estimation:

$$\hat{u}_s = \arg \max_u \sum_t w(s, t) \log p(v_t|u). \quad (3)$$

PolSAR data are damaged by speckle noise, described by a zero-mean complex circular Gaussian distribution [4] with a  $3 \times 3$  covariance matrix  $\mathbf{T} = \mathbb{E} \{ \mathbf{k} \mathbf{k}^\dagger \}$  where  $\mathbf{k}$  is a 3-dimensional complex vector and  $\dagger$  indicates the transpose complex conjugate. Usually, the vector  $\mathbf{k}$  is defined by  $\mathbf{k} = [S_{HH}, \sqrt{2}S_{HV}, S_{VV}]^t$  where  $S_{HH}$ ,  $S_{HV}$  and  $S_{VV}$  are the scattering coefficients associated to the three different transmitting and receiving polarizations: HH, HV and VV. The superscript  $t$  indicates the transpose of a vector. However, this vector can also be decomposed in the Pauli basis as  $\mathbf{k} = \frac{1}{\sqrt{2}}[S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV}]^t$  to better reflect polarimetric properties. Eventually, the WML estimate of the covariance matrix is given by:

$$\hat{\mathbf{T}}_s = \frac{\sum w(s, t) \mathbf{k} \mathbf{k}^\dagger}{\sum w(s, t)}. \quad (4)$$

#### 4. SETTING OF THE WEIGHTS

As mentioned in Section 3, the weights should approach the indicator function of the set of redundant patches. In order to consider the statistical nature of the observed image, we use the probabilistic criterion introduced in [3], where the weights are set to:

$$w(s, t) = \prod_k [Pr(v_{s,k}, v_{t,k} | u_{s,k} = u_{t,k}) Pr(u_{s,k} = u_{t,k})]^{1/h}. \quad (5)$$

In the following the pixels  $s, k$  and  $t, k$  will be denoted respectively by  $_1$  and  $_2$ . The first term  $Pr(v_1, v_2 | u_1 = u_2)$  reflects the likelihood to have identical (unknown) noise-free values with respect to the observed noisy image. A similar criterion has been applied in [5] to data damaged by additive white Gaussian noise. We extend here its definition as follows

$$Pr(v_1, v_2 | u_1 = u_2) = \left| \frac{d\Phi}{dv_1}(v_1) \right|^{-1} \left| \frac{d\Phi}{dv_2}(v_2) \right|^{-1} \int p(v_1 | u_1 = u) p(v_2 | u_2 = u) du \quad (6)$$

where the Jacobian terms are introduced to take into account the change of variables due to a mapping function  $\Phi$ . The mapping  $\Phi$  is introduced to obtain a dimensionless weight. Since the priors  $p(v_1 | u_1 = u)$  and  $p(v_2 | u_2 = u)$  are unknown, the authors in [3] assume the priors are uniform improper density functions. However, the integration is difficult to carry out over  $u$  while this quantity can instead be approached by the maximum likelihood similarity [6]:

$$Pr(v_1, v_2 | u_1 = u_2) \approx \left| \frac{d\Phi}{dv_1}(v_1) \right|^{-1} \left| \frac{d\Phi}{dv_2}(v_2) \right|^{-1} p(v_1 | u_1 = \hat{u}_{MV}) p(v_2 | u_2 = \hat{u}_{MV}) \quad (7)$$

where  $\hat{u}_{MV}$  is the maximum likelihood estimate over  $v_1$  and  $v_2$ . In the case of PolSAR data, the empirical estimate of the covariance matrix  $\hat{\mathbf{T}}_{MV}$  from only two observations  $v_1$  and  $v_2$  becomes singular, i.e. it cannot be inverted. To avoid this problem, we suggest to enforce  $\hat{\mathbf{T}}_{MV}$  to be diagonal. Thus by choosing  $\Phi : (|\mathbf{k}_{\cdot,1}|, |\mathbf{k}_{\cdot,2}|, |\mathbf{k}_{\cdot,3}|) \mapsto (|\mathbf{k}_{\cdot,1}|^{-1}, |\mathbf{k}_{\cdot,2}|^{-1}, |\mathbf{k}_{\cdot,3}|^{-1})$ , equation (7) gives:

$$Pr(\mathbf{k}_1, \mathbf{k}_2 | \mathbf{T}_1 = \mathbf{T}_2) \approx \left[ \frac{|\mathbf{k}_{1,1}| |\mathbf{k}_{1,2}| |\mathbf{k}_{1,3}| |\mathbf{k}_{2,1}| |\mathbf{k}_{2,2}| |\mathbf{k}_{2,3}|}{(|\mathbf{k}_{1,1}|^2 + |\mathbf{k}_{2,1}|^2)(|\mathbf{k}_{1,2}|^2 + |\mathbf{k}_{2,2}|^2)(|\mathbf{k}_{1,3}|^2 + |\mathbf{k}_{2,3}|^2)} \right]^2. \quad (8)$$

The second term  $Pr(u_1 = u_2)$  tries to measure the *prior* probability to have equal noise-free values at site  $_1$  and  $_2$ . In [7], the authors propose to use the Kullback-Leibler divergence on an estimate  $\hat{u}$  of  $u$  as a statistical test of the hypothesis  $u_1 = u_2$ . We have also observed good performance in practice of such a definition:

$$Pr(u_1 = u_2) = \exp \left[ -\frac{1}{T} SD_{KL}(\hat{u}_1, \hat{u}_2) \right]$$

where  $SD_{KL}(\hat{u}_1, \hat{u}_2) = \int (p(v|\hat{u}_1) - p(v|\hat{u}_2)) \log \frac{p(v|\hat{u}_1)}{p(v|\hat{u}_2)} dv \quad (9)$

where  $\hat{u}$  is an estimate of  $u$  and  $T$  a filtering parameter. For PolSAR data, the symmetric Kullback Leibler divergence between two zero-mean complex circular Gaussian distributions is given by

$$SD_{KL}(\hat{\mathbf{T}}_1, \hat{\mathbf{T}}_2) \propto tr \left( \hat{\mathbf{T}}_1^{-1} \hat{\mathbf{T}}_2 \right) + tr \left( \hat{\mathbf{T}}_2^{-1} \hat{\mathbf{T}}_1 \right) - 6. \quad (10)$$



**Fig. 1.** PolSAR images of Dresden (Germany) sensed by E-SAR ©DLR obtained from (a) the single-look complex HH SAR image, (b) the boxcar estimator and (c) the NL-PoSAR filter. The colorization is based on intensity and entropy-alpha decomposition.

The use of an estimate  $\hat{\mathbf{T}}$  provides an iterative algorithm which refines the weights at each iteration. Starting with the identity matrix at each site  $s$ , the weights  $w(s, t)$  are then computed between each sites  $s$  and  $t$  as defined by Equation 5. Then a new covariance matrix  $\hat{\mathbf{T}}_s$  is given by using Equation 4. The procedure is then repeated iteratively until convergence. During the iterations, it is important to enforce a minimum amount of smoothing. In our filter, we suggest to select when required, the ten most similar pixels according to the similarity between the patches.

## 5. EXPERIMENTS AND RESULTS

The NL-PoSAR filter has been applied successfully on three co-registered single-look complex SAR images (polarizations HH, HV and VV) of Dresden (Germany) sensed by E-SAR (L-Band). We use search windows of size  $21 \times 21$ , patches of size  $7 \times 7$  and the parameters  $h$  and  $T$  are set automatically as explained in [3]. The result is given on Figure 1 and compared to the original  $512 \times 512$  single-look complex HH SAR data and the result of the boxcar filter over a  $5 \times 5$  sliding window. The images are presented with a colorization based on an HSV (Hue Saturation Value) color space built from intensity and entropy-alpha ( $H-\alpha$ ) decomposition [8]. The proposed NL-PoSAR filter provides the best quality image. The noise is well reduced while the resolution is well preserved. Moreover the use of intensity and  $H-\alpha$  based color space shows that the scattering properties, such as interchannel information, seems well-preserved by NL-PoSAR. In the final article, numerical results obtained on synthetic data, such as the signal to noise ratio values, will be provided to demonstrate the efficiency of the proposed approach.

## 6. CONCLUSION

A non-local filter has been proposed to denoise polarimetric synthetic aperture radar images without significant loss of resolution. The proposed method searches iteratively the most suitable pixels to combine in order to produce the less biased noise-free estimate of the complex covariance matrix. The pixels are selected according to the joint similarity between the different channels of surrounding patches of noisy vectors and the joint similarity between patches extracted from pre-estimated noise-free covariance matrices. In the final paper, a more detailed description of the behavior of the proposed similarity criterion will be given. The obtained results demonstrate that the proposed method works well on L-Band E-SAR data with a good noise reduction without significant loss of spatial resolution. Moreover the interchannel information seems to be well preserved during the denoising procedure. These results will be completed with comparisons to other filters, such as [9] or [10], and quantitative criteria such as the signal to noise ratio values.

## 7. REFERENCES

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