**Quantifying the uncertainty in 35GHz+94GHz space-borne radar retrievals of rain**

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The main capability of the dual-frequency profiling radar proposed for deployment on the Aerosol/Cloud/Ecosystems (ACE) mission [1] is its ability to allow retrievals of vertical profiles of water content and mean hydrometeor size. At the center of this capability is the notion that measuring the radar reflectivity at two sufficiently different frequencies should allow the retrieval of two water variables. Since the measurements are made vertical-bin by vertical-bin, from the measurements of the vertical profiles of the pair of average reflectivity factors in each bin, one would retrieve the average water content and the average hydrometeor size in that bin. The main problems with this notion are

1) the non-linearity of the relations between the measured average radar reflectivity factor and the underlying variables that one wants to retrieve, namely the water content and the hydrometeor size;

2) the many-to-one nature of the forward relation between the underlying variables and the radiometric measurement: different pairs of mean water content and mean hydrometeor size may well produce the same pair of reflectivity factors, even in the absence of any measurement errors.

To quantify the first problem, we studied the case of liquid clouds and precipitation, and considered different Γ-distribution models (of the form $N(D)= N_0\, D^{\mu}\, e^{-\Lambda D}$ ) for the number of droplets of diameters $D$ per unit volume within a nominal 250m vertical resolution bin along the nadir beam of our radar. The non-linearity in the relation

$$F(N_0, \mu, \Lambda) = (Z_{35}, Z_{94})$$

between our condensed-water parameters and the corresponding radar reflectivity factors implies that a direct inversion would convert the inevitable white noise in the observations into a bias in the retrieved quantities, because a non-linear function of the mean of a variable is not equal to the mean of the non-linear function of that variable:

$$F^{-1}(Z_{35}+\text{whitenoise}, Z_{94}+\text{whitenoise}') \neq (\text{avg } N_0, \text{avg } \mu, \text{avg } \Lambda)$$

To isolate this effect from that of the many-to-one nature of the relations, we assumed $\mu$ is known exactly, and considered cases where the chosen values for the two remaining parameters guarantee that the forward relations, though non-linear, are one-to-one at least in the vicinity of our specific values. Figure 1 illustrates the assumptions, and figure 2 shows the results of the case where we assumed a homogeneous 3km-high column of 0.64mm/hr rain with drops whose mass-weighted mean diameter is 0.49mm. In this case, the 35GHz reflectivity factor is 15 dbZ, while the 94GHz reflectivity factor is 13 dbZ, guaranteeing, according to figure 1, that the rain-radar relations are locally 1-to-1. Figure 2 shows the results of deterministic retrievals that start with the same pair of reflectivity profiles, to which we added white noise with a standard deviation of 0.25db (left panel) or 0.3db (right panel).

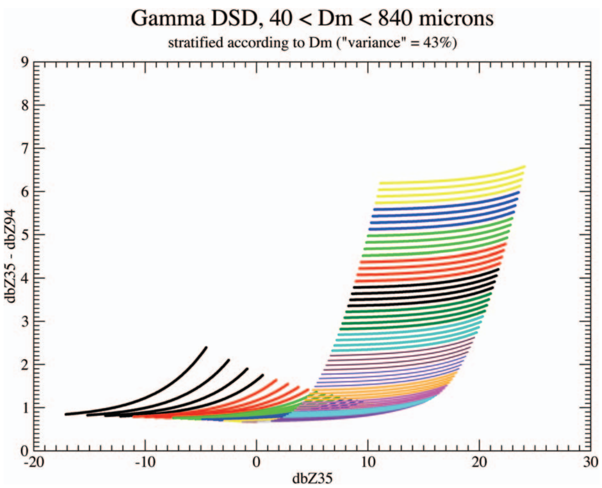


Figure 1: Joint scatter of $dbZ_{35}$ and $dbZ_{35}$-$dbZ_{94}$ as a function of the drop size distribution parameters, with each point colored according to the interval of values of the mass-weighted mean drop diameter $D_m$.

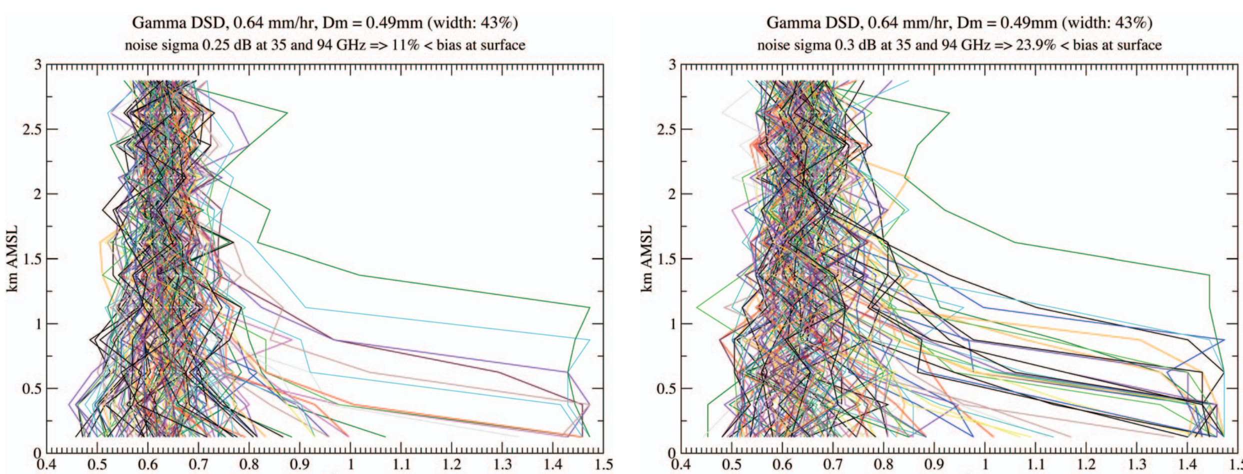


Figure 2: Retrievals for a constant-rain-rate column assuming 0.25db noise (left) or 0.3db noise (right)

Manifestly, with an increase of the white noise standard deviation from 0.25 to 0.3 db, the bias in the estimated rain rate at the surface increases from a non-negligible 11% to a quite large 24%.

The realistic case is more complicated since it combines the non-linearity with the many-to-one nature of the forward relations. Quantifying the severity of the problem by the relative uncertainty $\rho$ in the rain rate consistent with a given ($dbZ_{35}$, $dbZ_{35}$–$dbZ_{94}$) pair, it turns out that the problem is greatest (i.e. $\rho > 50\%$) when the reflectivity factors are such that $dbZ_{35}<11$ and $dbZ_{35}–dbZ_{94}<2$, which correspond to rain rates smaller than 0.7mm/hr with mass-weighted mean drop sizes between 70μm and 0.46mm (the retrieval uncertainty is smaller in the case of larger convective-sized drops – the upper-right portion of figure 1, as well as in the case of smaller cloud-sized droplets – the lower-left portion of figure 1).

The main implication is that the single-bin uncertainties are not negligible. To reduce them, vertical correlations (between bins) have to be physically characterized and subsequently enforced in the retrievals. Specifically, while the correlation between mean drop diameter $D_m$ and rain rate $R$ (or water content) is small, the values of these two quantities should be correlated vertically, and this correlation must be exploited to produce retrievals with a physically realistic uncertainty. Assuming a correlation coefficient of 0.15 between $\log D_m$ and $\log R$ ([2]), the first principal component of the vertical vector of $D_m/R^{0.15}$ can be derived from observations, and to the extent that it characterizes a large portion of the uncertainty in that ratio, it can be used to constrain realistically the uncertainty in a given column.

**References:**


[1] National Research Council, "Earth Science and applications from space: national imperatives for the next decade and beyond," *The National Academic Press*, 2007.

[2] Haddad Z. S. et al., 1997: A new parametrization of the rain drop size distribution, *IEEE Trans. Geosci. Rem. Sens.*, vol. 85, 532-539.