TOWARDS BAYESIAN ESTIMATOR SELECTION FOR QUIKSCAT WIND AND RAIN ESTIMATION

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1. INTRODUCTION

Wind and rain estimation over the ocean is possible using data provided by the QuikSCAT scatterometer. The QuikSCAT scatterometer measures the radar cross section or backscatter of the ocean and uses a model function to infer the most likely wind vector and rain rate to have produced the observed measurements [1]. It is estimated that rain effects 4 to 10% of all QuikSCAT observations. To account for these rain effects there are three slightly different estimation techniques which may be employed: wind-only[2], simultaneous-wind-rain [3, 4, 5, 6], and rain-only estimation [7]. The performance of each estimator is dependent on the underlying wind-rain conditions. As such, each estimation technique is best under certain backscatter conditions and no single technique is suitable for all conditions. However, by adaptively selecting the estimates most appropriate to the true conditions, performance can surpass that of any individual estimator. In this paper we introduce Bayes estimator selection, a technique whereby a single 'best' estimator can be selected for each case, and then apply the technique to QuikSCAT wind and rain estimation.

2. M-ARY ESTIMATOR SELECTION

M-ary Bayes estimator selection is a modification of Bayes decision theory which attempts to select a single 'best' estimate from M viable candidate estimates. In the ideal case, the 'best' estimate is the estimate which has minimum squared error to the true vector. For most interesting estimation situations the true vector is unknown. In this case it is not possible to determine the ideal solution so some approximation must be used. Bayes estimator selection is based upon the assumption that selecting the estimate with minimum expected squared-error is an appropriate approximation to the ideal minimum squared-error estimate.

The Bayes decision theory mechanism [8] is an ideal framework with which to construct such an estimator selection rule. The Bayes risk function for a true parameter distribution F_θ and a decision rule ϕ_j can be written as

$$r(F_\theta, \phi_j) = \int_\theta \sum_{i=1}^{M} L[\vartheta, \phi_j(\mathbf{x}_i)] f_{\mathbf{X}|\theta}(\mathbf{x}_i|\vartheta) f_\theta(\vartheta) d\theta \tag{1}$$

θ is a random variable representing the true conditions which has realizations ϑ and a pdf $f_\theta(\vartheta)$. $\mathbf{X}_i$ represents the observation random variable with realizations indicated by $\mathbf{x}_i$. $L[\vartheta, \phi_j(\mathbf{x}_i)]$ is a loss function representing the cost of the jth decision rule ϕ_j based on the observation $\mathbf{x}_i$ and true condition ϑ. $f_{\mathbf{X}_i|\theta}(\mathbf{x}_i|\vartheta)$ is the conditional pdf of the observation random variable $\mathbf{X}$ conditioned on the true parameter θ. M refers to the number of realizations of the observation random variable which contribute to a single decision.

This mechanism can be adapted for estimator selection by appropriately redefining each component in Eq. 1. For estimator selection the largest adjustment is that the observations $\mathbf{x}_i$ are in reality estimates of the realization of the parameter ϑ. Thus we exchange the notation of $\mathbf{x}_i$ for $\hat{\vartheta}_i$. This change is a fundamental difference from traditional Bayes decision theory and cannot be made lightly. In traditional Bayes decision theory the typical purpose is to make a decision about what the realization of the underlying parameter θ is, where ϑ is typically drawn from a discrete space. In this case, the purpose is to estimate the realization ϑ of the parameter θ which is a member of a continuous space. In essence we have fundamentally changed the meaning of the decision mechanism and adopt only the traditional mechanics for selecting between the previously determined estimates.

With this change in mind the decision rule $\phi_j(_i)$ can be interpreted as the decision to accept the estimate ϑ_j as the 'best' estimate based on the observation of the estimate ϑ_i. The interpretation of the conditional pdf must also be redefined. For estimator selection we can interpret the conditional pdf $f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta))$ to represent the probability that the estimate $\hat{\vartheta}_j$ has minimum squared-error given the true conditions.

Recall that the objective is to select a 'best' estimate, i.e. the estimate which has minimum expected-squared-error. This can be achieved by defining an appropriate loss function

$$L[\vartheta, \phi_j(\hat{\vartheta}_i)] = (\vartheta - \hat{\vartheta}_i)^T N(\vartheta - \hat{\vartheta}_i)(\kappa\delta_{ij} + \tau(1 - \delta_{ij})) \tag{2}$$

Here N is a diagonal normalization matrix to appropriately weight the components of the vector ϑ. κ and τ are also weighting terms to weight the loss function based on which estimate and which decision rule are used. κ determines the weight of the loss when the estimate is that selected by the decision rule ϕ_j and τ gives the weight of the loss when the estimate is not selected by the decision rule.

With the appropriately defined loss function the Bayes risk can be written after some simplification

$$r(F_\theta, \phi_j) = \int_\theta (\vartheta - \hat{\vartheta}_j)^T N(\vartheta - \hat{\vartheta}_j)(\tau + (\kappa - \tau) f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta)) f_\theta(\vartheta) d\vartheta \tag{3}$$

This definition helps interpret the weight coefficients κ and τ. When $\tau = \kappa = 0.5$ the Bayes risk represents the prior expected-squared-error of the estimate. When $\tau = 0$ and $\kappa = 1$ the Bayes risk represents the posterior expected-squared-error of the estimate. Thus the weight coefficients can be understood to weight the contribution of the prior and the posterior squared-error.

The estimate with minimum expected-squared-error is readily selected using Eq. 3 since the Bayes risk for this definition is the expected-squared-error. Thus selecting one of M candidate estimates as the 'best' estimate is simplified to choosing the estimate which has the minimum Bayes risk.

3. APPLICATION TO WIND AND RAIN ESTIMATION

Adopting the proposed Bayes estimator selection mechanism to QuikSCAT wind and rain estimation is relatively straightforward once the parts of Eq. 3 have been appropriately defined in the context of wind and rain estimation. For this case the parameter θ represents the wind and rain vector random variable. Each realization ϑ corresponds to a realization of a true wind and rain vector. The estimates of ϑ, $\hat{\vartheta}_i$, correspond to the wind-only, simultaneous wind-rain and rain-only estimates indexed by $i = 1, 2, 3$ respectively.

For this short paper we omit a thorough treatment of the prior distributions $f_{\hat{\theta}|\theta}(\hat{\vartheta}_j|\vartheta))$ and $f_\theta(\vartheta)$ for space considerations. The estimator selection technique is not particularly sensitive to a specific prior formulation as long as the priors generally represent the wind and rain distribution and the estimator performance. An empirically derived histogram has very similar performance to a more formal and sophisticated definition. We similarly omit a formal derivation of the normalization matrix N and the weighting coefficients τ and κ for space considerations in this paper. For reference $\tau = 0.26$, $\kappa = 0.74$, and N is a $M \times M$ diagonal matrix with $[1/50^2, 0, 1/150^2]$ as the diagonal elements. The weighting coefficients are dependent on a specified rain detection performance level. The normalization matrix N is defined such that the wind error is weighted by the inverse of the square of the maximum retrievable wind speed (50 m/s), direction error is ignored to simplify ambiguity selection, and rain error is weighted by the inverse of the square of an approximate upper bound on retrievable rain rate (150 km-mm/hr).

4. CASE STUDY

To demonstrate the potential utility for Bayes estimator selection in the context of QuikSCAT wind and rain estimation, we consider the following case study. Figure 1 shows the results of the estimator selection process for a single case study. First note the rain contamination of the wind-only estimates which is visible as high wind estimates corresponding to high TRMM PR measured rain rates. There is a similar effect in the rain-only estimates due to wind contamination in places where there is no rain. The simultaneous wind and rain estimates have more reasonable wind and rain estimates in raining conditions but have no wind estimates in many locations where there is no rain. The Bayes estimator selected wind and rain estimates demonstrate the strengths of each of the estimators while ameliorating their limitations. Thus the Bayes selected wind estimates have little rain contamination and have good performance in non-raining areas. The Bayes selected rain estimates correlate well with the measured TRMM PR rain rates and have relatively few missed detections of significant rain events.

5. CONCLUSIONS

To summarize Bayes estimator selection has the potential to improve wind and rain estimation techniques in the areas where conventional estimators have weaknesses. The technique can successfully identify and estimate rain contamination and identify areas where wind estimation is not possible due to dominating rain contamination. This leads to improved wind and rain estimation performance on a global scale and simultaneously gives insights into the limitations and possible improvements for future scatterometers.

6. REFERENCES

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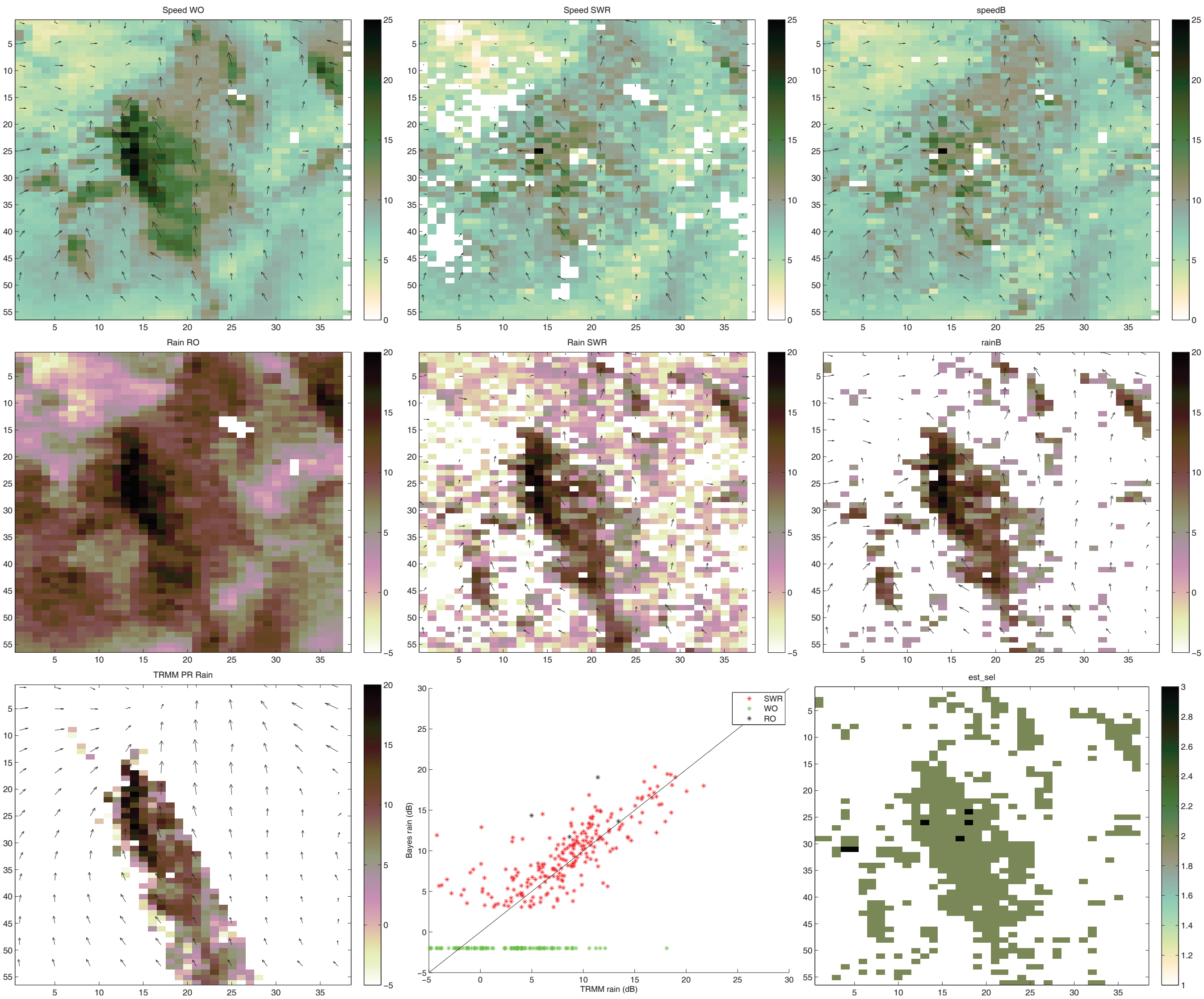

Fig. 1. Estimator selection results for a single case. Upper left: WO estimates. Upper center: SWR wind estimates. Upper right: Bayes selected wind estimates. Middle left: RO estimates. Middle center: SWR rain estimates. Middle right: Bayes selected rain estimates. Lower left: TRMM PR measured rain with NCEP wind vectors overlaid. Lower center: TRMM PR measured rain v. Bayes selected rain estimates. Lower right: Bayes estimator selections. Wind estimates are in m/s and rain estimates in dB km-mm/hr.