STATISTICAL PROPERTIES OF OCEAN SURFACE HEIGHTS USING REMOTE SENSORS WITH VARIABLE LINE OF SIGHT ANGLE.

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1. INTRODUCTION

The ocean wave motion data can be estimated by using aerial photographs of the Sun glint patterns, which show reflections of the Sun and sky light from the water and thus offer high-contrast ocean wave images.

Cox and Munk [1], [2] studied the distribution of glitter pattern in aerial photographs of the sea. They proposed that for constant and moderate wind speeds, the probability density function of the slopes is approximately Gaussian. In these circumstances, the ocean surface could be modeled as a Gaussian random process.

Álvarez-Borrego [3], [4], [5] derived the equation which describes the glitter pattern in one and two dimensions. With this glitter function we can find the variance of the intensities in the image from the variance of the surface slopes. It is then possible to obtain the relationship between the correlation function of the intensities in the image and the correlation function of the surface slopes.

Álvarez-Borrego [6] considered the problem of retrieving spatial information of the statistical properties of random rough surfaces from images via remote sensing. He obtained expressions relating the intensity variance in the image and the surface heights. He considered the detector located at the zenith for each point on the surface.

Here, we are considering a more real physical situation, where the detector can be located off the zenith and it is covering a certain range of angles (Fig.1). Thus, we obtained a new relationship between the variance of the surface slopes and the variance of the intensities in the image. Likewise, the correlation function of the surface slopes allows us to estimate the correlation function of the intensities in the image.

2. GEOMETRY OF THE MODEL

In figure 1, the surface ζ(x) is illuminated by a uniform incoherent source S and its image is formed in the detector D. The incidence angle, θ_s, represents the mean angle subtended by the source S, $(\theta_d)_i$ represents the angle

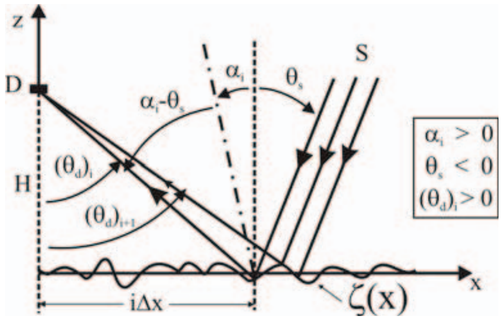

Figure 1. Geometry of the real physical situation.

subtended by the line of sight of the detector with the normal to the point i of the surface, and α_i is the slope angle at the surface point i. The acquired image is composed of bright and dark regions called glitter pattern. The glitter function can be expressed like:

$$B(M_i)=\mathrm{rect}\left[\frac{M_i-M_{oi}}{(1+M_{oi}^2)\frac{\beta}{2}}\right], \tag{1}$$

where

$$M_{oi}-(1+M_{oi}^2)\frac{\beta}{4}\le M_i\le M_{oi}+(1+M_{oi}^2)\frac{\beta}{4}, \tag{2}$$

$$M_i=\tan(\alpha_i) \qquad \text{and} \qquad M_{oi}=tan\left[\frac{\theta_s+(\theta_d)_i}{2}\right] \tag{3}$$

3. THE VARIANCES OF THE INTENSITIES IN THE IMAGE AND OF THE SURFACE SLOPES

The mean of the image, μ_I, may be written [8]:

$$\mu_I=\langle I(x)\rangle=\int_{-\infty}^{+\infty}B(M_i)p(M_i)dM_i=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\sigma_M(2\pi)^{1/2}}\int_{-\infty}^{+\infty}rect\left[\frac{M_i-M_{oi}}{(1+M_{oi}^2)\frac{\beta}{2}}\right]exp\left(-\frac{{M_i}^2}{2\sigma_M^2}\right)dM_i. \tag{4}$$

where B(M$_i$) is the glitter function, equation (1), and p(M$_i$) is the gaussian probability density function in one dimension.

Defining $a_i=M_{oi}-(1+M_{oi}^2)(\beta/4)$ and $b_i=M_{oi}+(1+M_{oi}^2)(\beta/4)$, we can write

$$\mu_I=\langle I(x)\rangle=\frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}\left[erf\left(\frac{b_i}{\sqrt{2}\,\sigma_M}\right)-erf\left(\frac{a_i}{\sqrt{2}\,\sigma_M}\right)\right]. \tag{5}$$

The variance of the intensities in the image, σ_I^2, is defined by [8]

$$\sigma_I^2=\int_{-\infty}^{+\infty}[B(M_i)-\mu_I]^2\,p(M_i)dM_i=\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{1}{2}\left[erf\left(\frac{b_i}{\sqrt{2}\,\sigma_M}\right)-erf\left(\frac{a_i}{\sqrt{2}\,\sigma_M}\right)\right]-\frac{1}{4N}\left[erf\left(\frac{b_i}{\sqrt{2}\,\sigma_M}\right)-erf\left(\frac{a_i}{\sqrt{2}\,\sigma_M}\right)\right]^2\right\}, \tag{6}$$

which is the required relation between the variance of the intensities in the image, σ_I^2, and the variance of the surface slopes, σ_M^2.

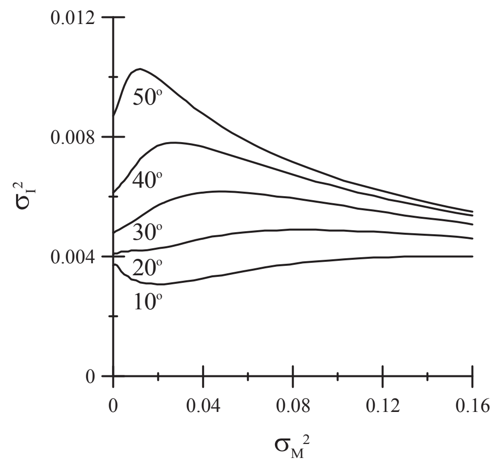

Figure 2. Theoretical relationship between the variance of the surface slopes with the variance of the intensities of the image.

The relationship between the variance of the surface slopes with the variance of the intensities of the image for different θ_s angles (10°-50°) is shown in figure 2. This relationship between σ_I^2 and σ_M^2 is different to the one proposed in Alvarez-Borrego [6] and Cureton et al. [7]. For higher incidence angles, the source image in the glitter pattern is wider, so variance is higher when the incidence angle increases.

4. CORRELATION FUNCTIONS OF THE IMAGE INTENSITIES AND OF THE SURFACE SLOPES

The relationship between the correlation function of the surface slopes $C_M(\tau)$ and the correlation function of the image intensities $C_I(\tau)$ is given by

$$C_I(\tau) = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{+\infty}\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{+\infty}\frac{B(M_{1i})\,B(M_{2i})}{2\pi\sigma_M^2\left[1-C_M^2(\tau)\right]^{1/2}}\exp\left[-\frac{M_{1i}^2+M_{2i}^2-2C_M(\tau)M_{1i}M_{2i}}{2\sigma_M^2\left[1-C_M^2(\tau)\right]}\right]dM_{1i}dM_{2i}. \quad (7)$$

Figure 3 shows the normalized correlation function of the image intensities $[C_I(\tau)]_n$ for σ_M=0.2121 and the same angles of the figure 2. Again, the relationship between $[C_I(\tau)]_n$ and $[C_M(\tau)]_n$ is different than the one proposed by Alvarez-Borrego [6] and Cureton et al [7]. For higher incidence angles, $[C_I(\tau)]_n$ is also higher.

5. CONCLUSIONS

We derive the variance of the surface slopes from the variance of the intensities of the image via remote sensing considering a glitter function given by equation (1) in the one-dimensional case and considering the more realistic physical model given by figure 1. We also discussed the determination of the correlation function of the surface

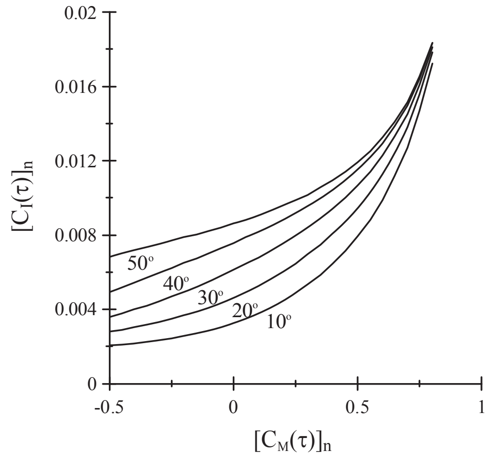

Figure 3. Relationship between the correlation function of the surface slopes and the correlation function of the intensities in the image.

slopes from the autocorrelation function of the intensities in the image. We think that these results present the most realistic case in a practical situation.

6. REFERENCES

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