# A NEW DIFFERENTIAL EVOLUTION ALGORITHM WITH COOPERATIVE COEVOLUTIONARY SELECTION OPERATOR FOR WAVEFORM INVERSION

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#### 1. INTRODUCTION

Waveform inversion is a potential tool for geophysics to identify the subsurface structure. The gradient-based waveform inversion algorithms have got some successful applications, but they are limited by the nonlinearity of the inverse problem and need a good guess for initial model. The global optimization methods were first used by earth scientists more than 30 years ago. Their advantage is that they needn't to compute the gradient and can work well without good guess of initial model. Evolutionary algorithms (EA) are a set of global optimization methods which seek optimal solutions by mimicking the seemingly random natural processes by which species evolve. They have been applied with success to Earth sciences by many authors. For example, Groot-Hedlin and Vernon use the improved evolutionary programming to estimating the layered velocity structure [2], Hong and Sen use multiscale genetic algorithm to invert the earth model [5].

But the conventional EA often lose their effectiveness and advantages when applied to high-dimensional and complex problems. Cooperative coevolution (CC) was proposed by Potter and Jong [1] to solve high-dimensional optimization problems through problem decomposition, but it only work well with the separable problem. Yang, Tang and Yao have improved the original CC approach for the weak nonseparable problem by introducing a grouping based decomposition strategy [6]. However, there still has not a general efficient approach for solving the high-dimensional strong nonseparable problem, e.g., high-dimensional waveform inversion.

In this paper, we propose an improved differential evolution (DE) for the prestack surface seismic waveform inversion. DE is a floating-point encoding EA, which has been shown to be a simple yet powerful algorithm for global optimization [3, 4]. By incorporating some attractive concepts of CC and Simulated Annealing (SA) into the selection operator of DE, the new DE is effective for high-dimensional waveform inversion problem. Another novel feature of this paper is that the local fitness functions are introduced to evaluate the subcomponents of the high-dimensional individual. We refer the DE with a cooperative coevalutionary selection operator as DE-CCS.

### 2. WAVEFORM INVERSION WITH DE-CCS

## 2.1. Differential Evolution and Cooperative Coevolution

According to the paper of Storn and Price, DE contains NP D-dimensional individuals (or vectors) in each generation:  $\bar{x}_i^G = \left\{x_{i,1}^G, x_{i,2}^G, \cdots, x_{i,D}^G\right\}$ , G denotes the generation and  $i = 1, 2, \cdots, NP$ . The main operations of DE may be summarized as follows [3, 4]. Mutation: creating the mutant individual  $\bar{v}_i^G$  by adding a weighted difference vector between two individuals to a third individual. Crossover: creating the trial individual  $\bar{u}_i^G$  by getting its variables from  $\bar{x}_i^G$  and  $\bar{v}_i^G$  randomly. Selection: selecting next generation  $\bar{x}_i^{G+1}$  according to the fitness values of the parent individual  $\bar{x}_i^G$  and the trail individual  $\bar{u}_i^G$ .

Cooperative coevolution is proposed to solve high-dimensional problems [1, 6]. The main idea of CC is: First decomposing the high-dimensional objective vector into smaller subcomponents. Then 'evolve' each subcomponent using a certain EA respectively in multiple cycles, until the termination condition is satisfied. Via this divide-and-conquer method, CC is able to solve many separate problems effectively.

## 2.2. DE-CCS Algorithm for Waveform Inversion

Given the observed data from a seismic survey (or nature earthquake), simply stated, waveform inversion aims to find the best earth model that consistent with the data. In this paper, we pose the waveform inversion as to minimize the following objective function (or fitness function):

$$f(\mathbf{m}) = \left\| \mathbf{d}_o - \mathbf{d}_c(\mathbf{m}) \right\|_2, \tag{1}$$

where  $\mathbf{d}_o$  is the observed data,  $\mathbf{d}_c(\mathbf{m})$  is the computed data with guessed earth model parameters  $\mathbf{m}$ ,  $\| \bullet \|_2$  represents the  $L_2$  norm.

Waveform inversion is a highly nonlinear problem. While the earth model contains a large number of layers, e.g., several hundreds, it usually fails to find a near-real earth model with the common global optimization methods. CC can't be directly used to waveform inversion problems for two reasons. First, waveform inversion is a strong nonseparable problem, for which there are tight interdependence among variables. Second, the fitness evaluation is very computationally expensive for waveform inversion.

Note that, whether DE or CC, they use a uniform global fitness function to evaluate all the variables in an individual. However, one individual which has worse global fitness than other individuals may contain some better variables. How can we keep these partial good variables to the next generation? For this purpose, and for our waveform inversion problem, we improve the DE with a new selection operation. The new selection operation comprises two steps.

At the first step, adopting the concept of CC, the individuals are decomposed into some subcomponents. Each subcomponent contains a few successive layers of earth model parameters. We then assign a local fitness function for each subcomponent by adding a corresponding short time-window to the prestack surface seismic data after

normal moveout (NMO) correction. Figure 1 shows an extreme example: in which each subcomponent contains only one layer. The local fitness function for the jth subcomponent can be written as:

$$Lf_{j}(\mathbf{m}) = \left\| \left[ \mathbf{d}_{o} - \mathbf{d}_{c}(\mathbf{m}) \right] \cdot \mathbf{win}_{j} \right\|_{2}, \tag{2}$$

where  $\mathbf{win}_j$  is a time window with its midpoint at the jth layer. Although the jth local fitness function is also indirectly affected by the parameters above the jth subcomponent, it is mainly affected by the jth subcomponent. In some sense, it provides a criterion for evaluating the jth subcomponent. We produce a mid offspring  $\bar{y}_i^{G+1}$  one subcomponent by one subcomponent according to the local fitness of the individual  $\bar{x}_i^G$  and the trail individual  $\bar{u}_i^G$ . At the second step, considering the interdependence among subcomponents, coevolution is needed, the final offspring is selected according to the fitness values of the parent individual  $\bar{x}_i^G$  and the mid offspring  $\bar{y}_i^{G+1}$ . But the probability concept for the selection operator of SA is added to this step.

$$\bar{x}_{i}^{G+1} = \begin{cases} \bar{y}_{i}^{G+1} & \text{if } f(\bar{y}_{i}^{G+1}) < f(\bar{x}_{i}^{G}) \text{ or } rand(0,1) < p_{s}, \\ \bar{x}_{i}^{G} & \text{otherwise,} \end{cases}$$
  $i = 1, 2, \dots, NP,$  (3)

where  $p_s$  is a constant factor between 0 and 1, usually less than 0.2.

## 2.3. Application Example

Here, the new DE-CCS is applied on a pre-stack seismic waveform inversion problem to estimate the 1-D acoustic model parameters, i.e., the wave velocity V and density  $\rho$ . Although it is a synthetic example, the model parameters are taken from the real well logs. While computing the observed seismic data, the model is divided into 237 layers in depth, with an interval of dz=10 m. Source signature is a 35 Hz Ricker wavelet. In the implementation of inversion, the model is parametrized as functions of seismic traveltime, with interval of dt=8 ms. then there are 205 velocities and 205 densities needed to estimate in this example. A 30% variation superimposed on the low frequency trends of well logs defines the search space (Fig. 2). Population size is NP=40. For comparison, we use DE-CCS and convention DE to estimate the V and  $\rho$  with the same number of fitness evaluations (1200\*40). The inverted optimal V are showed in Fig. 2. We can see that, the V inverted with DE-CCS is consistent with the well log, and the V inverted with convention DE does not converge to the well log. Similar results have also been obtained on the parameter  $\rho$ , but it is not shown in this paper.

### 3. CONCLUSIONS

This paper proposed an improved global inversion algorithm, DE-CCS, for the waveform inversion problems. The example with well logs shows that DE-CCS is effective for high-dimensional waveform inversion. In each iteration step DE-CCS needs twice times of fitness evaluations than DE, but DE-CCS performs better than DE with the same total number of fitness evaluations. In the future work, we would like to compare DE-CCS with

some other global optimization methods, and find a strategy for choosing the length of the subcomponents and the length of the time-window.

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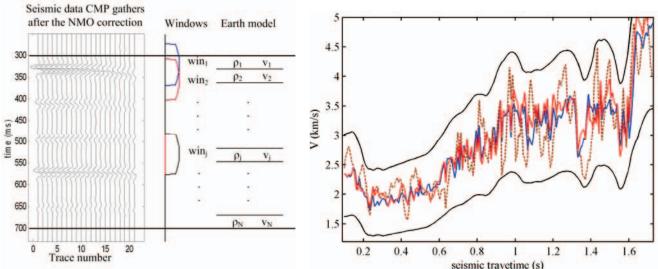


Fig. 1. The left volume shows the prestack surface seismic data after normal moveout (NMO) correction. The right volume give an example of the decomposition of the high-dimensional individual, here each subcomponent contains only one layer. The mid volume shows the corresponding time-window for each subcomponent.

Fig. 2. The inverted optimal velocity V with DE-CCS (red line) and the inverted optimal V with DE (brown line), they are compared with the well log (blue line). The dark lines define the search space.