1. INTRODUCTION

Bathymetry is an important variable in scientific and operational applications. Passive remote sensing offers a cost-effective alternative to both LiDAR and echo sounding survey for measuring water depth with the advantage that data are collected synoptically over large areas. Lyzenga [1][2] proposed bathymetry algorithms for both a single wavelength band and a pair of wavelength bands. The common practice is to fit and calibrate a single global model for the entire image scene. Namely, model parameters are assumed constant throughout the scene. The performance of conventional global models is limited when the water quality and bottom type vary spatially in the scene. To address the inadequacy of the conventional global models, we propose a locally adaptive log-linear model to better estimate the water depth. Although the general form of the model is the same, model parameters are optimally determined within a local area. Our analysis shows that locally adaptive log-linear models can effectively address the problem of spatial heterogeneity of water quality and bottom type and hence provide more reliable and accurate bathymetric estimates for more complex coastal waters. Also, we demonstrate that the model parameters can be spatially interpolated and mapped for interpreting spatial variation of bottom type and water quality.

2. CONVENTIONAL GLOBAL REGRESSION INVERSION MODEL

Based on Beer’s Law that the intensity of light is attenuated exponentially with depth traveled through the water column, Lyzenga [1][2] and Philpot [3] proposed a simple radiative transfer model for optically shallow waters. Assuming that the ratio of bottom reflectances between two spectral bands is constant for all bottom types within
a given scene, Lyzenga [1][2] derived a bathymetric inversion model for two (and/or multiple) spectral bands as follows:

\[ z = a_0 + \sum_{i=1}^{N} a_i \ln[L(\lambda_i) - L_\infty(\lambda_i)] \]  

(1)

Where \( a_i \) (\( i=0,1,\ldots, N \)) are the constant coefficients, \( N \) is the number of spectral bands, \( L(\lambda_i) \) is the remote sensing radiance after atmospheric and sunglint corrections for spectral band \( \lambda_i \), and \( L_\infty(\lambda_i) \) is the deepwater radiance for spectral band \( \lambda_i \). \( z \) is depth. Equation (1) is referred to as the log-linear inversion (or deepwater correction) model, which has been most widely used for estimating water depths from optical multi-spectral remote sensing imagery.

This model is based on two assumptions. The first assumption is the uniform water-quality throughout the scene. If the attenuation coefficient remains constant, the water depth will be insensitive to water quality change. The second assumption is that the ratio of bottom reflectance in two bands is the same for all bottom types. However, in most situations, this ratio changes with bottom types and water quality and that assumption is invalid for bathymetric mapping within the entire scene. Moreover, attenuation coefficient of the water varies with the changing water depth in reality. Therefore, one global regression equation across entire spatial data sets is not proper for the bathymetry retrieval in the turbid water column and/or the heterogeneous bottom types.

3. INVERSION MODELS BASED ON LOCALLY ADAPTIVE REGRESSION

To address spatial nonstationarity, the locally adaptive regression model is applied based on the principle of geographically weighted regression [4]. Given a set of \( n \) observations and for each observation, the underlying model for local regression is:

\[ z_i(x, y) = \beta_0(x, y) + \sum_{j=1}^{m} L_{ij} \beta_j(x, y) + \epsilon_i(x, y) \]  

(2)

where \( z_i \) is the observed water depth of the \( i \)th target point; \( i = 1, 2, \ldots, n \); \( L_{ij} \) represents the radiance of the \( j \)th neighboring point for the \( i \)th target point; \( \beta_0, \beta_1, \beta_2, \ldots, \beta_m \) are regression coefficients; \( \epsilon_i \) is the random error term. \((x, y)\) are geographic coordinates of target point. Regression coefficients correspond to every target point and one regression model is fitted locally for each target point. A kernel window with a distance is drawn around the location \( i \) and all observed points in the window are all used for the local regression (figure 1b). Coefficients \( \beta_0, \beta_1, \beta_2, \ldots, \beta_m \) are estimated by a weighted least-square method. This allows a point-by-point calibration of the procedures with consequent improvement in their adaptability to local situations. Thus, more efficient predictive models can be obtained which more accurately estimate the dependent variables.
Additionally, the computation of different regression models for each image pixel allows the estimation of spatially variable accuracy statistics.

Regional regression models are also performed in our study. In regional regression models, a regression equation is fitted in each region specifically (figure 1a) while in local adaptive regression model, a regression equation is fitted for the target point \( i \) using points located in the moving window with the radius \( r \) (figure 1b).

![Figure 1. Regional regression model vs Local adaptive regression model](image.png)

**4. APPLICATION RESULTS**

A case study has been conducted to test and evaluate the local adaptive regression algorithm for bathymetry inversion. Our study area is part of the south shore of Kauai Island, Hawaii. A natural-color aerial photo is used in our analysis. We used the blue and green bands of the natural-color aerial photo and a set of reference depth points from the SHOALS to calibrate the bathymetric inversion models and derive a bathymetric grid for the entire scene.

<table>
<thead>
<tr>
<th>Residual</th>
<th>Global regression</th>
<th>Regional regression</th>
<th>Local regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average over-estimate</td>
<td>0.96 m</td>
<td>0.89 m</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Average under-estimate</td>
<td>-1.23 m</td>
<td>-1.01 m</td>
<td>-0.16 m</td>
</tr>
<tr>
<td>Average overall</td>
<td>1.11 m</td>
<td>0.80 m</td>
<td>0.15 m</td>
</tr>
</tbody>
</table>

Table 1 shows that local regression offered much more accurate depth estimates than both global and regional regressions. The average residual of local regression is about 0.15 m while it is 1.11 m for global regression and 0.80 m for regional regression. Overall, as shown in our assessment, the local adaptive regression offers much better estimations of water depth than global regression and regional regression. Finally, as shown in figure 2, the distribution of residual by local adaptive regression tends to be random. Overall, as shown in our
assessment, the local adaptive regression offers much better estimations of water depth than global regression and regional regression.

Figure 2: distribution of residuals by local adaptive regression

5. CONCLUSIONS

A locally adaptive regression model is proposed to estimate the shallow water depth. Instead of a single global model, we apply locally adaptive regression for each location, allowing the local bathymetric estimation. Locally adaptive regression models account for spatial variations in water quality and bottom types in the study area and gives the localized patterns in the estimated bathymetry. We demonstrate that the estimation accuracy of water depth from locally adaptive regression models is higher than that of global and regional regression model.

6. REFERENCES