RANDOM NOISE SAR BASED ON COMPRESSED SENSING

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1. INTRODUCTION

Random Noise SAR [1] is a kind of radar imaging with noise or noise-like transmit signal, usually the transmit signal can be regarded as additive white gauss noise (AWGN). Compressed sensing (CS) [2] describes a method for reconstructing sparse signals from relatively few measurements, typically well below the number expected from the requirements of the Shannon/Nyquist sampling theorem. In CS theory shows that the measurement matrix whose entries are drawn independently from certain distributions satisfies the Restricted Isometry Property (RIP) [3] very well, and guarantees exact recovery of the targets which are sparse or sparse in any basis with high probability. In this paper we analyzed the RIP of the measurement matrix in the random noise SAR system, and we have presented some random noise SAR simulations via compressed sensing.

2. PRELIMINARIES

In many cases, we are interested in signals x which is sparse of N complex values. Thus, we assume that only $k < N$ entries are significant in magnitude in signal x. In this case compressed sensing methods deal with recovering the k most significant entries of using the measurements given by Φx, where Φ the measurement matrix satisfies the RIP. We first recall some definitions:

Definition 1 [4] *The discrete L^P norm of any array A is defined as:*

$$\|A\|_p \leq \left(\sum_{j=0}^{N-1} |A(j)|^p \right)^{\frac{1}{p}}$$

Definition 2 [3] *A matrix Φ satisfies the* $\mathrm{RIP}(N, K, \delta)$ *of order K with constant $\delta \in (0,1)$ if*

$$(1-\delta)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1+\delta)\|x\|_2^2$$

Hold for all x such that $\|x\|_0 \leq K$.

Given a matrix Φ satisfies the $\mathrm{RIP}(N, K, \delta)$ with $\delta \in (0,1)$ sufficiently small, a near optimal K- term approximation can be obtained for Φx using linear programming [5], or regularized orthogonal matching pursuit (ROMP) variants [6].

Definition 3 [4] *The Hadamard Product of an $M \times N$ matrix $A = [a_{ij}]$ and an $M \times N$ matrix $B = [b_{ij}]$ is record as $A \odot B$, it is also an $M \times N$ matrix, it is defined as:*

$$A \odot B = [a_{ij} b_{ij}]$$

Lemma 1 *Let Φ by a $M \times N$ matrix with elements drawn i.i.d according to $N(0,1)$ and let $A \in C^{M \times N}$ be a matrix with normalized columns. Furthermore, let $u \in C^N$ is an arbitrary vector with N entries. Let $\delta \in (0,1)$ be given. Then the matrix $\Psi = A \odot \Phi$ satisfies*

$$E\left(\|\Psi u\|_2^2\right) = \|u\|_2^2$$

and

$$P\left(\left|\|\Psi u\|_2^2 - \|u\|_2^2\right| \geq \delta \|u\|_2^2\right) \leq 2e^{-M\delta^2/8}$$

Lemma 2 *Let Φ by a $M \times N$ matrix with elements drawn i.i.d according to $N(0,1)$ and let $A, B \in C^{M\times N}$ are matrices with normalized columns. Furthermore, let $x = \begin{bmatrix} u & v \end{bmatrix} \in C^{2N}$ is an arbitrary vector with first N entries is u and the last N entries is v. Let $\delta \in (0,1)$ be given, then the matrix $\Psi = \frac{1}{\sqrt{2}} \begin{bmatrix} A \odot \Phi \\ B \odot \Phi \end{bmatrix}$ satisfies*

$$E\left(\|\Psi x\|_2^2\right) = \|x\|_2^2$$

and

$$P\left(\left|\|\Psi x\|_2^2 - \|x\|_2^2\right| \geq \delta \|x\|_2^2\right) \leq 2e^{-M\delta^2/4}$$

Theorem 1 *Let Φ by a $M \times N$ matrix with elements drawn i.i.d according to $N(0,1)$ and let $A \in C^{M\times N}$ are matrices with normalized columns. If*

$$M \geq C_1 K \log\left(\frac{N}{K}\right)$$

Then the matrix $\Psi = A \odot \Phi$ satisfies the RIP of order K with probability exceeding $1-3e^{-C_2 M}$, where C_1 and C_2 are constants that depends only on the desired RIP constant δ.

The proof of these conclusions can be completed with reference to the ralated works in [7],[8],[9].

3. RANDOM NOISE SAR

3.1. Random Noise SAR Model

An elementary block diagram of a random noise SAR system model based on compressed sensing is shown in Figure 1. There are mainly three ways used in the A/D sampling, they are random sampling, random demodulation [10], and downsampling. Many reconstruction algorithms can be used in compressed sensing processing, such as linear programming [5], ROMP [6] and the laplace method [11].

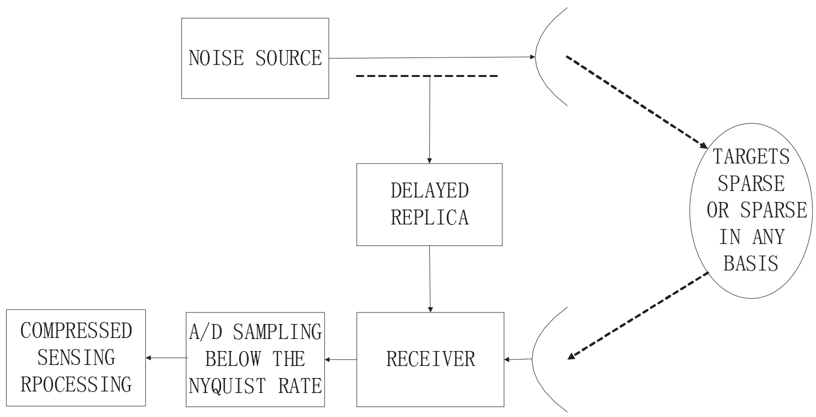


Fig. 1. The Random Noise SAR Model Based on Compressed Sensing

For a relatively short correlation time, the random noise SAR received signal model can be simplified to the form:

$$x_R(t) = \sum_{i=1}^{N_T} A_i x_T(t - \frac{2R_i(t)}{C}) e^{-j\frac{4\pi}{\lambda}R_i(t)} + \xi_t(t)$$

where N_T is the number of sparse targets, $x_T(t)$ is the transmit signal, C is the speed of light, λ is the wavelength of the transmit signal, A_i denotes the complex amplitude of the $i-\text{th}$ targets echo, $R_i(t) = \sqrt{R_i^2 + (Vt)^2}$ its range (V is the speed of the random noise SAR platform) and $\xi_t(t)$ is AWGN. For simplicity, only noiseless condition is concerned in our discussion.

3.2. Application of Compressed Sensing

The Radar Cross Section (RCS) used in random noise SAR is a sampled 2D spatial spectrum with the size $K \times L$. The RCS $\sigma[m, n]$ of the target scene can be converted into a very long vector by stacking its columns. That is,

$$\sigma = \begin{bmatrix} \sigma_1^T & \cdots & \sigma_L^T \end{bmatrix}^T$$

where

$$\sigma_n = \begin{bmatrix} \sigma[1, n] & \cdots & \sigma[K, n] \end{bmatrix}^T$$

Similarly, let $\hat{y}$ be vector constructed by $M \times N$ received data the same way,

$$\hat{y} = \begin{bmatrix} \hat{y}_1^T & \cdots & \hat{y}_N^T \end{bmatrix}^T$$

where

$$\hat{y}_n = \begin{bmatrix} \hat{y}[1, n] & \cdots & \hat{y}[M, n] \end{bmatrix}^T$$

The relation between $\hat{y}$ and σ is given by

$$\hat{y} = \Phi\sigma$$

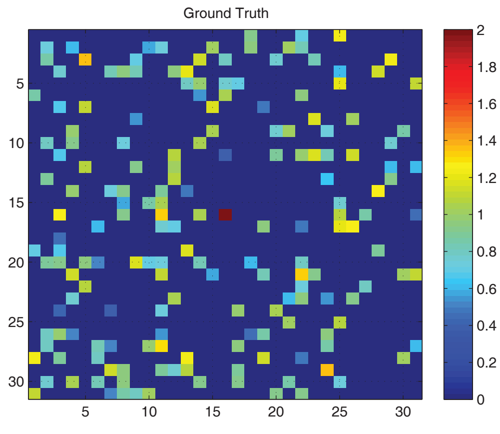
where Φ is a $MN \times KL$ matrix whose $i, j-\mathrm{th}$ element is

$$[\Phi]_{i,j} = x_T(t_i - \frac{2R_j(t_i)}{C})e^{-j\frac{4\pi}{\lambda}R_j(t_i)}$$

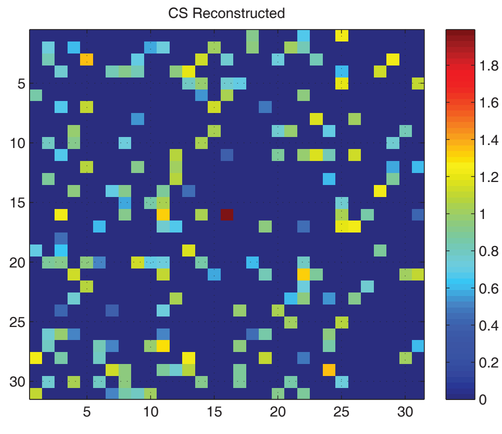
In random noise SAR, the transmit signal $x_T(t)$ is AWGN, so we can see Φ as a random matrix with elements drawn i.i.d according to $N(0, \sigma^2)$. For simplicity, we can let $\sigma = 1$, and we have that $\|\exp(-j4\pi R(t)/\lambda)\|_2^2 = 1$. If we let $A = \Phi/\sqrt{MN}$, then A satisfies the conditions stated in Theorem 1, using the Theorem 1's result, we know that the measurement matrix satisfied the RIP very well, and can be reconstructed by compressed sensing.

4. SIMULATION RESULT

In this section, we provide some simulation results to illustrate the claims of Theorem 1. The simulation is to show that the sparse target scene can be exactly recovered from random noise SAR echo by compressed sensing. The results are shown in Fig 2.



(a)The Truth Targets



(b)Recovered by compressed sensing

Fig. 2. Signal recovery from noise radar echo.

In Figure 2, (a) is the target scene of size 31×31 who was generated at random with random magnitude between 0 and 2 with 200 nonzero targets, (b) is the result recovered by compressed method with 30×30 echoes by row and column random sampled from the 85×160 echo data using the laplace method [11], and its 2-norm relative error $\|\hat{x} - x\|_2/\|x\|_2 = 0.0125$, sampling rate $\eta = (30 \times 30)/(85 \times 160) = 0.0662$,the result show that compressed sensing can be successfully used in random noise SAR signal processing.

5. DISCUSSION

In this paper, we have studied the problem of recovery the sparse targets from random noise SAR echo. We have proofed that the random noise SAR measurements matrix satisfied the RIP very well with high probability, and our simulations demonstrate that we can recover sparse signals exactly at the number of measurements required in the theory. The main result is that the sparse or sparse in any basis targets can be exactly recovered from the random noise SAR echo with high probability. The result can also be used in noisy condition. Some simulations in noisy conditions are still in processing.

6. REFERENCES

[1] D.C. Bell and R.M. Narayanan, "Theoretical aspects of radar imaging using stochastic waveforms," *IEEE Trans. on Signal Processing*, vol. 49, no. 2, pp. 394–400, Feb 2001.

[2] D. Donoho, "Compressed sensing," *IEEE Trans. on Inform. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr 2006.

[3] E. J. Candes and T. Tao, "Decoding by linear programming," *IEEE Trans. on Inform. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec 2005.

[4] Horn, Roger A, Johnson, and Charles R, *Matrix Analysis*, Cambridge University Press, 1985.

[5] E. J. Candes and T. Tao, "Near optimal signal recovery from random projections: Universal encoding strategies?," *IEEE Trans. on Inform. Theory*, vol. 52, no. 12, pp. 5406–5425, Dec 2006.

[6] D. Needell and R. Vershynin, "Uniform uncertainty principle and signal recovery via regularized orthogonal matching pursuit," *Found. Comput. Math.*, vol. 9, no. 3, pp. 317–334, Apr 2009.

[7] S. Dasgupta and A. Gupta, "An elementary proof of the johnson-lindenstrauss lemma," *Tech. Report 99-006,U.C. Berkeley*, Mar 1999.

[8] R. G. Baraniuk, M. A. Davenport, R. DeVore, and M. B. Wakin, "A simple proof of the restricted isometry property for random matrices," *Constructive Approximation*, vol. 28, no. 3, pp. 253–263, Dec 2008.

[9] J. N. Laska, M. A. Davenport, and R. G. Baraniuk, "Exact signal recovery from sparsely corrupted measurements through the pursuit of justice," *Proc. Asilomar Conf. on Signals Systems and Computers*, Nov 2009.

[10] E.J. Candes and M.B. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, Mar 2008.

[11] S. D. Babacan, R. Molina, and A. K. Katsaggelos, "Bayesian compressive sensing using laplace priors," *IEEE Trans. on Image Processing*, vol. 19, no. 1, pp. 53–63, Jan 2010.