# MULTIBASELINE GRADIENT AMBIGUITY RESOLUTION TO SUPPORT MINIMUM COST FLOW PHASE UNWRAPPING


*Marie Lachaise* (1), *Richard Bamler* (1)(2)

*Fernando Rodriguez Gonzalez* (2)

(1) German Aerospace Center (DLR)
Remote Sensing Technology Institute
Oberpfaffenhofen, 82234 Wessling, Germany

(2) Technische Univesität München (TUM)
Remote Sensing Technology
Arcisstrasse 21, 80333 Munich, Germany


## 1. INTRODUCTION

The TanDEM-X Mission will start in the first half of 2010. Its primary objective is the generation of a consistent global Digital Elevation Model (DEM) with an unprecedented accuracy. The whole land mass will be mapped with two different baselines in order to reduce the difficulty of phase unwrapping while achieving the required accuracy. Phase unwrapping is a crucial step to obtain this high quality DEM.

The method we propose here combines both Minimum Cost Flow (MCF, [1]) and Maximum Likelihood Estimation (MLE, [2] [3]). Firstly, MLE is used to solve phase gradient ambiguities on a pixel-by-pixel basis. The required search interval for gradient MLE is much smaller than the one for phase MLE. Thus computation is considerably faster and more robust. Secondly, phase unwrapping of the most accurate interferogram is performed with the MCF algorithm. Since gradient MLE has already solved or reduced the ambiguity error in gradient estimates, errors related to its estimation are reduced. Moreover, MCF introduces the overall conservative condition on the gradient, compensating the locality of the MLE stage. As a consequence, the advantages of both MCF and MLE are efficiently combined into a single robust framework.

## 2. MULTIBASELINE GRADIENT AMBIGUITY RESOLUTION TO SUPPORT MCF ALGORITHM

### 2.1. Multibaseline Gradient Maximum Likelihood Estimation

The gradient MLE consists of combining the gradient estimates of two or more interfergrams $\{\psi_l\}_{l\in\{1,\ldots,L\}}$ in order to reduce the gradient ambiguity. Gradients $\hat{\nabla}\psi(i,k)$ are estimated by computing partial derivatives of $\psi(i,k)$ and wrapping them back if they exceed $\pm\pi$. Their probability density function $pdf(\nabla\phi)$ is [4]

$$pdf(\nabla\phi)[i,k] \propto pdf(\phi;\gamma,L)[i+1,k] * pdf(-\phi;\gamma,L)[i,k]. \tag{1}$$

where $pdf(\phi;\gamma,L)$ is the *pdf* of an interferogram sample [4] and $*$ stands for convolution.

Once one of the interferograms has been selected as reference $\left(l=1\right)$, the gradient distributions of the other interferograms may be scaled by the ratio of baselines $a_l = B_1/B_l$, where $B_l$ is the baseline of the interferogram $l$. The multibaseline gradient likelihood function or *joint pdf* of the gradient is then

$$pdf(\nabla\phi_1, \nabla\phi_2, ..., \nabla\phi_N)[i,k] = \prod_{l}^{N} pdf(\phi_l a_l; \gamma_l, L)[i+1,k] * pdf(-\phi_l a_l; \gamma_l, L)[i,k]. \tag{2}$$

Since it is very unlikely to have a very high gradient, the search interval for the maximum is then reduced to three cycles ($\nabla\phi_1 \in [-3\pi, 3\pi)$) of the reference interferogram i.e. the one with the smallest baseline. As a result, the processing is much faster than usual MLE on the phase. In fig. 1 two cases are illustrated, both in terms of single and joint gradient pdf. Depending on the difference of the acquired gradients, the number of significant peaks of the resulting distribution may considerably vary (see fig. 2).

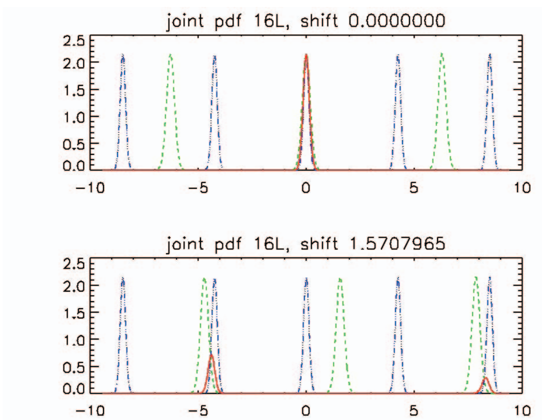


**Fig. 1**. Example of *pdf* (dashed) and *joint pdf* (solid) for two different configurations of the gradients *pdf* for $\gamma = 0.8$

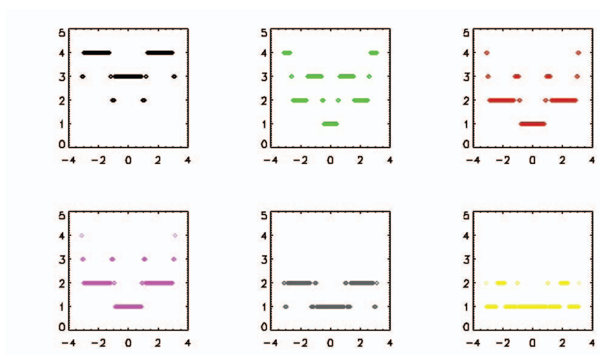

**Fig. 2**. Expected numbers of significant peaks according to the difference between both gradients for different $\gamma$ (from left to right, first line: 0.2, 0.4, 0.6; second line: 0.7, 0.8, 0.9)

The multibaseline gradient estimates do not exactely correspond to any of the ambiguities of the original interferograms. In order to allow the application of the MCF in the second stage of the algorithm, gradient estimates are rounded to their nearest ambiguities.

### 2.2. Minimum Cost Flow algorithm supported by unwrapped gradients

The MCF approach solves the following global minimization problem

$$\min_{d_i d_k} \left\{ \sum_{i,k} c_i(i,k)|d_i(i,k)| + \sum_{i,k} c_k(i,k)|d_k(i,k)| \right\}. \tag{3}$$

In the usual MCF approach, $d_i(i,k)$ and $d_k(i,k)$ are the residue fields and have values equal to $0, -2\pi, 2\pi$. They are used to correct the gradient estimate, making it conservative.

In our approach, new residue fields are calculated with the help of the unwrapped gradients from section 2.1. Since the interval of possible gradient estimates has been extended, their values can be now integer multiples of $2\pi$.

Finally, adapted cost functions $c_i(i,k)$ and $c_k(i,k)$ have been developped.

### 2.3. Cost functions

The application of the MCF algorithm for phase unwrapping has been traditionally enhanced by the use of cost functions which introduce information about coherence and local topography [1]. This weighting of the L1 norm is used here as a further connection between MLE and MCF. Given the locality, variability and quantized nature of the MLE correction, an adapted cost function derived from a quality estimator is required.

The following properties are desired. On the one hand, the presence of two peaks in the joint pdf (fig. 1 right) should have a lower cost than a single peak (fig. 1 left). Moreover, the more symetrical the distribution between the two peaks is, the lower the cost should be. The same principle has to be extended to any number of peaks. This last point is important, since depending on the coherence, the number of peaks may be considerably high (see fig. 2). On the other hand, the cost should reflect the value of the coherence, since it incorporates rich information on both phase quality and terrain structure, as it has been proven in single interferogram phase unwrapping.

A quality indicator of the gradient estimate is the spread of the resulting distribution. In order to exclusively account for the shape of the overall joint pdf, we normalize it with its maximum and then calculate the second moment around the value of the peak. Concretely, the quality indicator is the following

$$q[i,k] = \sqrt{\frac{\sum \left( pdf(\nabla\phi_l)[i,k](\nabla\phi - argmax(pdf(\nabla\phi_l)[i,k]))^2 \right)}{max(pdf(\nabla\phi_l)[i,k])}}. \tag{4}$$

The derived cost function is given by $c[i,k] = A * (B - log_{10}(q[i,k]))$ where the terms A and B are used for scaling and to assure that $\forall i,k \; c[i,k] > 0$. Its evaluation for different coherences and phase differences between the two gradients values is given in fig. 3. We verify the desired dependency on coherence and the clear link with the number of peaks (see fig. 2).

## 3. RESULTS

Multibaseline interferograms have been simulated using a DEM obtained from a repeat-pass TerrarSAR-X (TSX) interferogram and real TSX geometrical parameters. Hence our data is realistic regarding geometrical aspects, but without any atmospheric artefacts. Moreover, the level of noise has been controlled.

We simulated interferograms with two different baselines. The first interferogram has a height of ambiguity of 40.1 m/cycle and the second one of 27.0 m/cycle, analogous to TanDEM-X operational configuration. Search interval for gradient MLE is three cycles of the interferogram which is taken as reference. Multibaseline gradient estimation has been performed in order to remove the gradient ambiguity for each interferogram (see fig. 4a).

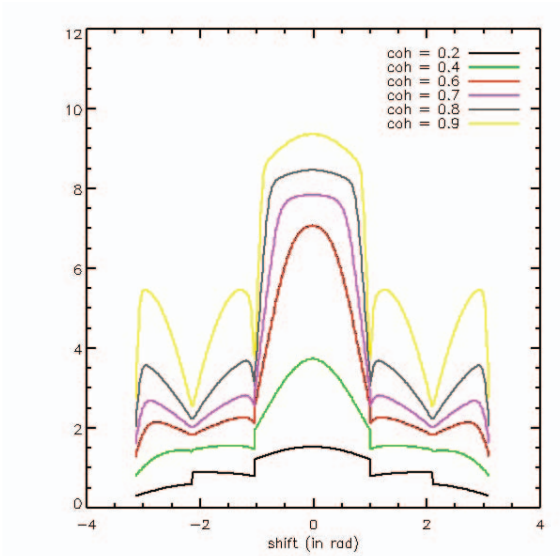


**Fig. 3**. Variation of the cost function $c[i,k]$ according to the shifts between the two gradients for different coherences (0.2, 0.4, 0.6, 0.7, 0.8, 0.9).

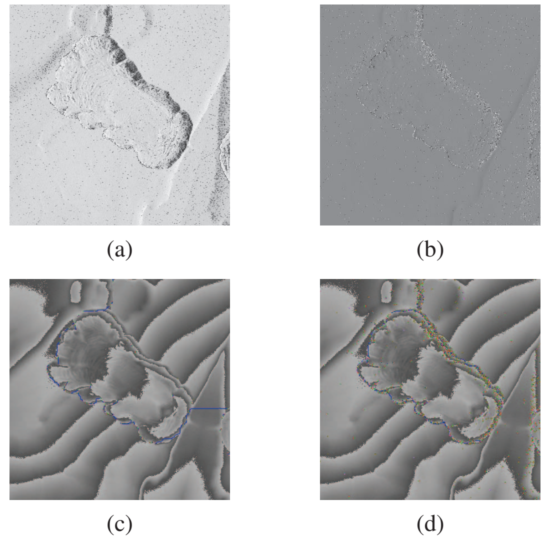


**Fig. 4**. Results obtained with simulated data from a TSX DEM ($\gamma \approx 0.8$): (a) dual-baseline unwrapped gradient in range, (b) derived costs obtained from the *pdf* distributions, (c) residues and branch-cuts obtained from the MCF algorithm without unwrapped gradients support and (d) residues and branch-cuts obtained from the MCF algorithm without unwrapped gradients support

The unwrapped gradients in range and azimuth are used as inputs to MCF. New residue fields are obtained (fig. 4c and 4d). It can be observed that most of the long branch-cuts are successfully removed. Hence the resulting unwrapped phase exhibits less errors. Concretely, in fig. 4c, there is a long and obviously erroneous branch cut in the lower part. It has been efficiently corrected by our approach (fig. 4d).

## 4. REFERENCES

[1] M. Eineder, M. Hubig, and B. Milcke, "Unwrapping large interferograms using the minimum cost flow algorithm," in *Geoscience and Remote Sensing Symposium Proceedings*. IEEE, 1998, pp. 83–87.

[2] M. Eineder and N. Adam, "A maximum-likelihood estimator to simultaneously unwrap, geocode, and fuse sar interferograms from different viewing geometries into one digital elevation model," *Transactions on Geoscience and Remote Sensing*, vol. 43, pp. 24–36, January 2005.

[3] G. Ferraiuolo, V. Pascazio, and G. Schirinzi, "Maximum a posteriori estimation of height profiles in insar imaging," *Geoscience and Remote Sensing Letters*, vol. 1, no. 2, pp. 66–70, April 2004.

[4] R. Bamler, N. Adam, G.W. Davidson, and D. Just, "Noise-induced slope distortion in 2-d phase unwrapping by linear estimators with application to sar interferometry," *Transactions on Geoscience and Remote Sensing*, vol. 36, pp. 913–921, May 1998.