

# OPTIMAL ALGORITHMS FOR SPACEBORNE ALTIMETER

*B.Y. Grishechkin (bg1722@aol.com), A.I. Baskakov (baskakovai@mpei.ru)*

Moscow Power Engineering Institute (Technical University)

## 1. INTRODUCTION

Space altimeters are used for global world ocean remote sensing more than 30 years. It helps to analyze sea currents variations as well to improve climate prediction and to resolve oceanographic, geodesic, geological, geophysical problems, etc.

## 2. ANALISIS OF ECHO

Applying experimentally well approved phenomenological model of sea surface radio signal reflecting and analytically analyzing the echo signal it was shown that the echo is a subject of fast and slow fluctuations. The fast fluctuations of echo signal come about because of RA-emitted radio wave leading edge is crossing sea surface facets, which provides random facets echoes initial phases when summing at the RA antenna when received. The slow fluctuations accord to the reflecting area increasing during the sensing pulse duration. It was shown [2] that the echo fast fluctuations correlation interval at the input of the RA receiver is inversely proportional to the compressed pulse duration (almost during the whole duration) therefore de-chirp technique of receiving may be used as an optimal in the signal to noise ratio criteria meaning. The echo slow fluctuations calculation [3] allowed to prove the selection of sensing pulse repetition interval. By and large in [2] it was shown that two-dimensional correlation function of RA echo at the matched filter (MF) output can be presented as a product of two independent functions

$$R(t, \tau) = \text{Re}[P_m \rho(\tau) P_n(t) \exp(-j\omega_0 \tau)], \quad (1)$$

where  $P_m$  – maximum power of the echo at the output of the MF,  $P_n(t)$  – normalized by  $P_m$  averaged form of the power of the signal at the MF output,  $\rho(\tau)$  – a fast changing function (in comparison with  $P_n(t)$ ) differing from the chirp auto-correlation function (with accounting of side lobes correction) by an exponent,  $\omega_0$  – center frequency of RA sensing pulse.

Theoretically the RA receiver can be based on a MF, processing the echo in the time domain, or based on a de-chirp scheme, processing the echo in the frequency domain. These two schemes are equal in theory, but in practice the second way is more adequate. In the time domain the MF output signal power can be presented [2] as

$$P_n(t) = \exp\left[\frac{b^2 \mu^2}{4} - bt\right] \cdot \left[0,5 - \Phi_1\left(\frac{b\mu}{2} - \frac{t}{\mu}\right)\right], \quad (2)$$

where  $b$  – parameter depending on the antenna beam width, antenna pointing error and sea surface roughness;  $\mu$  – according to the sea surface wave height parameter supposing that facets ordinates probability density is Gaussian

$$\mu = \sqrt{\frac{1}{\pi \Delta f_s^2} + 2 \left( \frac{2\sigma_z}{c} \right)^2} \quad (3)$$

$c$  – light velocity;  $\sigma_z$  – sea surface facets standard deviation, which can be recalculated into the significant wave height (SWH [1, 4])  $H_{1/3} = 4\sigma_z$ ;  $\Phi_1(x)$  – probability integral divided by 2.

When processing the echo in the frequency domain the averaged power spectrum of the signal at the de-chirp mixer output accounting side lobe correction by weighting window can be calculated as Fourier transform of the two-dimension correlation function (1) and after normalizing by maximum value of power spectral density  $S_m$  can be presented as [5]

$$S_n(f) = \exp\left[\frac{b^2\mu^2}{4} - \frac{bf}{k}\right] \cdot \left[0,5 - \Phi_1\left(\frac{b\mu}{2} - \frac{f}{k\mu}\right)\right], \quad (4)$$

where  $k$  – frequency slope in the sensing pulse and the frequency axis ( $f$  – axis) is shifted by the intermediate frequency value  $f_{\text{int}}$ .

It is the averaged power form of the echo at the MF output as well as the averaged power spectrum density at the de-chirp mixer output what can be called altimeter waveform. The vehicle height above the sea surface, SWH, sea surface backscattering coefficient  $\sigma_0$  and antenna pointing error accord to the offset and the shape of the waveform [2], therefore they can be revealed by processing the waveform. Practically the most performance demanding part of the processing is fulfilled by on-Earth centers.

### 3. DETECTION ALGORITHM

According to the theory of statistical radiolocation there must be a detection process before measuring. By synthesizing an optimal detector in [6] the logarithm of likelihood function for RA was presented

$$\Lambda[\xi(t)/\tau_0] = C_0 - \frac{1}{2N_0} \int_0^T \xi^2(t) dt + \frac{1}{2N_0} \int_0^T \frac{q(t)}{1+q(t)} \left| \int_{-\infty}^{\infty} \xi(\tau) e^{j\omega_0\tau} \dot{h}_f(t-\tau) d\tau \right|^2 dt, \quad (5)$$

where  $\xi(t)$  – input signal,  $\dot{h}_f(t-\tau)$  – pulse response characteristic of the MF,  $q(t)$  – current ratio of the echo power to the power spectral density of the RA receiver noise ( $N_0$ ) at the MF output,  $T$  – duration of the echo,  $C_0$  – a constant independent of the echo.

### 4. ESTIMATION ALGORITHMS

It is the discriminator what is the key unit of any measuring device. The discriminator forms a voltage or a code according to the difference of a parameter true value being measured and its estimate. The main task for RA is measuring the vehicle current height above the sea surface.

In papers [1, 4] it was proposed to estimate SWH by recalculating the slope of the waveform (3) or (4) which is evidently sensitive to the standard deviation of sea surface facets ordinates. But there is no exact

mathematical synthesis of according algorithm in literature. By and large the output signal of optimal discriminator (in the meaning of the least square error of the estimated parameter) is determined by the measuring parameter derivative of the likelihood function logarithm (5). The optimal discriminator algorithm is being synthesized while other parameters of the waveform are fixed. So when SWH is being measured other parameters (current altitude, antenna pointing error, signal to noise ratio) are assumed to be fixed. When current altitude is being measured other parameters (SWH, antenna pointing error, signal to noise ratio) are assumed to be fixed. Then other parameters variation influence is assessed. As for SWH mathematically it is suitable to synthesize  $\mu$  value (see eq.3) discriminator instead of SWH value, which can be recalculated into standard deviation of sea surface facets ordinates  $\sigma_z$  and then into SWH as mentioned above.

It can be shown that for processing in the frequency domain that discriminators has the following characteristics

$$\overline{U_{df}(\delta f)} = S_m \int_{-\infty}^{\infty} S_n(f) v_{cf}(f + \delta f, \mu) df, \quad (6)$$

$$\overline{U_{d\mu}(\delta \mu)} = S_m \int_{-\infty}^{\infty} S_n(f, \mu) v_{r\mu}(f, \mu + \delta \mu) df, \quad (7)$$

where the discriminators reference functions  $v_{cf}(f + \delta f, \mu)$ ,  $v_{r\mu}(f, \mu + \delta \mu)$  depend on parameters  $b$ ,  $\mu$ ,  $k$  and signal to noise ratio in the bandwidth of one filter of the spectrum processing unit corresponding to the maximum value of RA waveform

$$q_m = \frac{S_m}{N_0 \Delta F_c T}, \quad (8)$$

where  $\Delta F_c$  is inversely proportional to the correlation interval of the fast fluctuations mentioned above.

Equations (6) and (7) were used to analyze the discriminators fluctuation characteristic and to calculate the potential accuracy of measurements, see fig. 1 and fig.2, the following values were used for the plotting: vehicle orbit height  $h = 1000$  km,  $\Delta f_s = 320$  MHz,  $\tau_p = 102.4$   $\mu$ s,  $q_m = 20$  db.

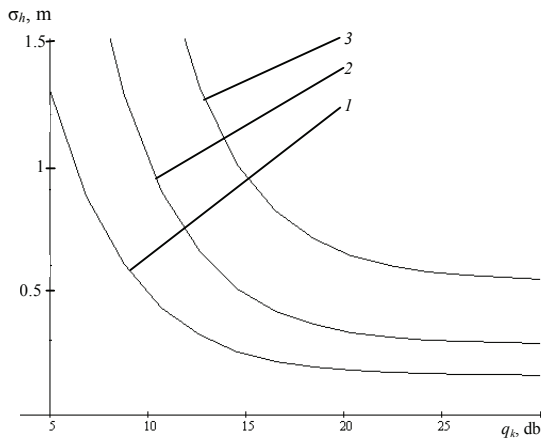


Fig. 1. Potential accuracy of height single measurement  
1 -  $\sigma_z = 0.2$  m; 2 -  $\sigma_z = 1$  m; 3 -  $\sigma_z = 4$  m

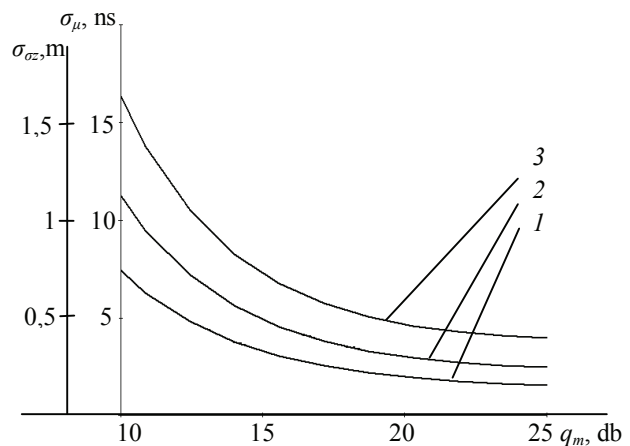


Fig. 2. Potential accuracy of parameter  $\mu$  and  $\sigma_z$  single measurement  
1 -  $\sigma_{z-tr} = 1$  m; 2 -  $\sigma_{z-tr} = 2$  m; 3 -  $\sigma_{z-tr} = 4$  m

Usually homogeneous sea surface roughness area is about 100 km or more, therefore it is adequate to average SWH measurements during 4 seconds or so. If the pulse repetition frequency is 1 kHz and  $\sigma_{z_{tr}}$  is from 1 m to 4 m and signal to noise is 20 db then averaging during 4 s can provide less than 1 cm error of  $\sigma_z$  measurement, i.e. 4 cm error for SWH.

## 5. CONCLUSION

This paper and our previous papers allow concluding the following resume:

1. Systematic errors of optimal RA discriminators for altitude and SWH measurements are closely coupled, because every reference function parameter error results in measured parameter error.
2. Good results of using optimal discriminator for target parameter can be achieved only if using measurements of other parameters: antenna pointing error (e.g. by waveform decay assesment) and signal to noise ratio (e.g. by auto gain control system of RA receiver). They should be used to calculate the reference function of optimal discriminator for accurate SWH measuring. Averaging time intervals for these parameters should be chosen in accordance with independent considerations.
3. Optimal discriminator demands compensation of noise component of the discriminator output signal, which can be provided by processing of RA input “signal” when there is no echo (before emitting next pulse).
4. Sea wave height accuracy (as well as altitude accuracy), can not be reduced less than a limit determined by the natural irreducible fluctuations of RA echo caused by random character of sea surface reflection.

## 6. REFERENCES

- [1] D.B. Chelton, E.J. Walsh, J.L. MacArthur “Pulse Compression and Sea Level Tracking in Satellite Altimetry”, Journal of atmospheric and oceanic technology, Vol. 6. – P. 407-438, 1989.
- [2] A.I. Baskakov, Ka Min Ho, N.A. Vazhenin, B.Y. Grishechkin “Correlation function analysis of precise space altimeter echo”, Proceedings of universities in Russia. Radio electronics. – Issue 1. – pp. 53-62 (in Russian), 2007.
- [3] Ka Min Ho, A.I. Baskakov “Selection of pulse repetition frequency in high-precision oceanographic radar altimeters”, IEEE Geoscience and Remote Sensing Letters, Vol. 4, pp. 345–348, 2007.
- [4] D.B. Chelton, J. Ries, B. Haines, L. Fu, P. Callahan “Satellite Altimetry and Earth Sciences: A handbook for techniques and applications”, San Diego: Academic Press. – 131 p, 2001.
- [5] A.I. Baskakov, N.A. Vazhenin, K.N. Morozov “Comparison of information signal processed using time- and frequency-domain methods in oceanographic precision radar altimeters”, Earth. Obs. Rem. Sens. – Vol.16. – P. 449-455, 2000.
- [6] B.Y. Grishechkin, A.I. Baskakov “Echo detection characteristics of precise space born oceanographic radar altimeter”, Aerospace devices engineering. – №5. – pp. 37-40 (in Russian), 2007.