A TEST STATISTIC FOR HIGH RESOLUTION POLARIMETRIC SAR DATA CLASSIFICATION

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1. INTRODUCTION

The polarimetric clutter obtained with recent SAR systems tends to have non-Gaussian characteristics. One of the most general and elegant non-Gaussian noise model is provided by the so-called Spherically Invariant Random Vectors (SIRV). This paper proposes a method for classification of high resolution polarimetric SAR data, based on a statistical test of equality of covariance matrices. It is organized as follows : Section 2 presents the statistical framework. Section 3 describes the proposed method. Then, results on experimental data are presented in Sect. 4.

2. STATISTICAL FRAMEWORK

A SIRV [1] is a compound Gaussian process defined as the product of a multivariate circular Gaussian process and a scalar random variable:

$$\mathbf{k} = \sqrt{\tau}\,\mathbf{x} \tag{1}$$

where τ, a positive random variable, called *texture*, whose Probability Density Function (PDF) is unknown and $\mathbf{x}$ is a complex circular zero-mean Gaussian m-vector with covariance matrix $\mathbf{T} = E\left[\mathbf{x}\mathbf{x}^H\right]$, called *speckle* where $E[.]$ denotes the statistical expectation. For POLSAR data, the polarimetric diversity is modeled by the speckle $\mathbf{x}$ containing the 3 polarization channels HH, HV and VV, i.e. $m = 3$ and the random variation of the power from cell to cell corresponds to the texture τ.

To ideally estimate the covariance matrix of a pixel (i, j), represented by its target vector $\mathbf{k}$, one needs several realizations of the pixel at different times. As it is impossible for SAR images, a spatial neighborhood of the pixel $(\mathbf{k_1}, ..., \mathbf{k_N})$ is required for the estimation process. To test the equality of a population covariance matrix, $\mathbf{T}$ and a known matrix, $\mathbf{T}_\omega$, the classical hypothesis test is defined as:

$$\begin{cases} H_0 : \mathbf{T} = \mathbf{T}_\omega \\ H_1 : \mathbf{T} \neq \mathbf{T}_\omega \end{cases} \tag{2}$$

In the classical Gaussian model, the Maximum Likelihood Estimator (MLE) of the covariance matrix is called the Sample Covariance Matrix (SCM):

$$\widehat{\mathbf{T}}_{SCM} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{k}_i \, \mathbf{k}_i^H \tag{3}$$

Under SIRV assumption, the covariance matrix can be estimated thanks to ML theory. Considering a deterministic texture, Gini *et al.* derived in [2] the exact ML estimate, solution of the implicit equation:

$$\widehat{\mathbf{T}}_{FP} = \frac{m}{N} \sum_{i=1}^{N} \frac{\mathbf{k}_i \, \mathbf{k}_i^H}{\mathbf{k}_i^H \, \widehat{\mathbf{T}}_{FP}^{-1} \, \mathbf{k}_i} \tag{4}$$

Existence and uniqueness of the above equation solution $\widehat{\mathbf{T}}_{FP}$, the Fixed Point (FP) estimate, have been investigated in [3], and its statistical properties (consistency, unbiasedness and asymptotic Gaussianity) have been studied in [4]. In practice, it is obtained by the associated recursive algorithm which converges whatever the initialization (see for details [3]).

3. M-DISTANCE

In [5], Lee *et al.* proposed an algorithm to classify pixels in a polarimetric SAR image, based on the statistical distribution of the covariance matrix. This algorithm used a distance measure, called the Wishart Distance, derived from the Wishart distribution of the covariance matrices. We propose to use a novel distance measure derived from a statistical test of equality of covariance matrices, the Box's M-test[6]. As discussed in [7], the test, originally developed for the real case, can be extended to the complex case. The test statistic is given by :

$$t = \frac{|\widehat{\mathbf{T}}_1|^{\frac{\nu_1}{2}} |\widehat{\mathbf{T}}_2|^{\frac{\nu_2}{2}}}{|\widehat{\mathbf{T}}_t|^{\nu_t}}$$

where ν_i is the degree of freedom of $\widehat{\mathbf{T}}_i$, $\nu_t = \nu_1 + \nu_2$ and $\widehat{\mathbf{T}}_t$ is the pooled sample covariance matrix defined by:

$$\widehat{\mathbf{T}}_t = \frac{\nu_1 \widehat{\mathbf{T}}_1 + \nu_2 \widehat{\mathbf{T}}_2}{\nu_1 + \nu_2}$$

As the exact distribution of t is rather cumbersome, Box proposes the following approximation for the distribution of u :

$$u = -2(1 - c_1) \ln(t) \sim \chi^2 \left(\frac{1}{2} m(m+1) \right)$$

where $c_1 = N \left(\dfrac{2m^2 + 3m - 1}{12\,(m+1)} \right)$ and $\chi^2(a)$ denotes the χ^2 distribution with a degrees of freedom. This approximation holds for Wishart-distributed matrices. Pascal *et al.* proved that the FP estimate is asymptotically Wishart-distributed with $\nu = (m/m+1)N$ degrees of freedom in [4]. The χ^2 approximation can therefore be used for the FP estimate.
We use the test statistic u from Eq. (3) as a distance measure in the following algorithm:

1. Initially classify the image into 8 areas using the H/α decomposition.

2. Select the first class of the H/α decomposition (the class situated in the top-right corner of the H/α plane) as the first

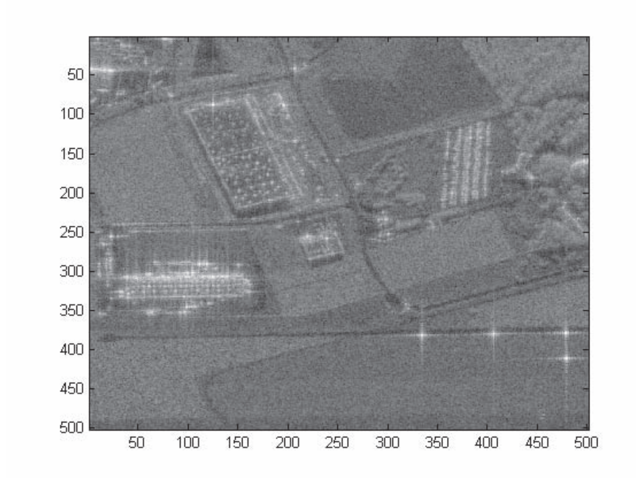


Fig. 1. Span image of the Brétigny area, France

class of the classifier. We have found that chosing a different class for the initialization had little to no influence on the end result.

3. For each pixel, compute the M-distance between its covariance matrix and each class center using Eq. (3) (in the first iteration, there is only one class center).

4. If the minimum distance is lower than the threshold given by $\chi^2\left(0.999, \frac{1}{2}m(m+1)\right)$, classify the pixel in the corresponding class. Else, put the pixel in the rejection class.

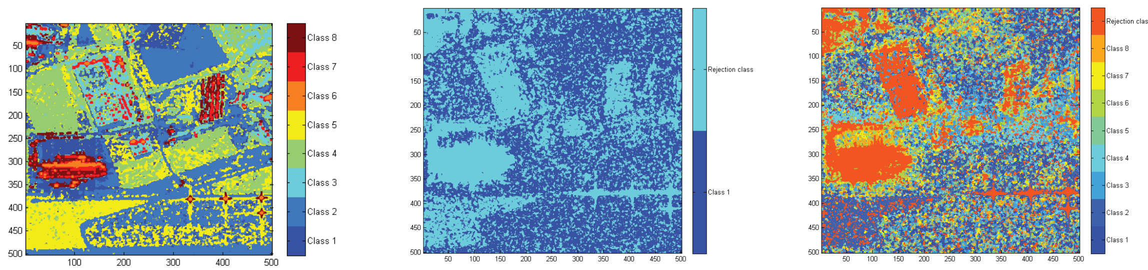
5. Once all pixels are classified, define the rejection class as a new class and compute its class center.

6. Repeat until there are 8 classes and a rejection class.

4. APPLICATION ON EXPERIMENTAL DATA

Experimental data were acquired in X-band by the ONERA RAMSES system in the area of Brétigny, France, with a spatial resolution of approximately 1.5 meter in range and azimuth, and a mean incidence angle of 30°. Fig. 1 is an image of the power (or *span*) of the Brétigny area. One can clearly distinguish two large buildings on the left side of the image, a parking lot on the right side, two small buildings in the middle, an urban area on the top-left corner and 4 bright spots on the lower-right corner, corresponding to trihedral corners. Fields and forested areas constitute most of the remaining part of the area. The algorithm from Sect. 3 has been applied on these data. The results can be seen below, on Fig. 2.

Fig. 2(a) show the results of the classical Wishart Classifier on our data, using the SCM. While some areas are well separated from the rest (namely, the fields in yellow, green and blue), the buildings areas contain many classes. Fig. 2(b) shows the results of the classifier after the first iteration. There are only 2 classes : the first class of the classifier and the rejection class. The features mentioned above are, for the most part, clearly identifiable on this image and they all belong to the rejection class. Fig. 2(c) shows the results of the classifier after 8 iterations. There are 9 classes, including the rejection class. The number of pixels contained in the rejected class has been reduced from 46% to 23%. When comparing Fig. 2(b) and Fig. 2(c), one can associate the detected class 1 with Gaussian clutter model with the class 9 detected with the SIRV clutter model. Regarding the classes labeled from 2 to 8 in Fig. 2(b), it is rather difficult to provide a direct subjective interpretation.



(a) (b) (c)

Fig. 2. Results of the Wishart Classifier and of the classifier using the M-distance

5. CONCLUSION

In this paper, authors proposed a novel distance measure to classify polarimetric covariance matrices. Results of the classical Wishart Classifier have been compared to our algorithm. For the final paper, we propose to interpret the classes objectively in terms of the Touzi physical parameters [8]. This way, the polarimetric information contained in each class can be evaluated for this data set.

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