PHASE RETRIEVAL IN SAR INTERFEROGRAMS USING DIFFUSION AND INPAINTING

A. Borzì, M. Di Bisceglie, C. Galdi, L. Pallotta, S. L. Ullo

University of Sannio, Piazza Roma 21, I–82100 Benevento, Italy

1. INTRODUCTION AND RELATED WORK

The main objectives of SAR interferometry are the generation of high-quality and high-resolution digital elevation models (DEMs) and earth–surface deformation maps. Many wanted and unwanted factors are responsible for phase variations, such as the topography, the possible deformations, the flat–earth effect, the atmosphere, the track error, the image misregistration, the thermal noise, the baseline decorrelation and so on. Refer to [1] and [2] for a detailed description of processing steps necessary to built interferograms from SAR images and for the different techniques used to reduce noise and improve interferograms quality. One of the most critical steps in interferometric SAR processing is phase unwrapping because the accuracy of this step is affected in a severe way from many phenomena (i.e. foreshortening, layover, shadowing, high temporal and/or high spatial baseline).

A number of unwrapping algorithms for SAR interferograms have proliferated over the past ten years, such as the Cut–Line (CL) algorithm [3], the Region–Growing (RG) algorithm [4], the Weighted Least Squares (WLS) algorithm [5].
It has been shown that the RG algorithm successfully unwraps low-coherence regions, better than the other two algorithms. It works across and around low-coherence regions making fewer ambiguity-level errors and resulting in a more accurate connecting of the high-coherence regions [6].
The CL algorithm is more prone to unwrapping errors because it cannot avoid linking the erroneous pairs of residues when the residue density is high. Also, the WLS algorithm results in unwrapping errors because it tries to fit the unwrapped phases to a smooth model in low-coherence areas, which is not valid in areas of significant topographic changes. A synthesis approach combining the cuts algorithm with a least–squares solution appears instead to offer greater spatial coverage with less distortion in many instances [7].
In 1996 Costantini [8] proposed a new branch–cut based phase unwrapping algorithm, that minimizes the total weighted length of branch cuts in the image, also known as the Minimum Cost Flow (MCF) problem. The critical point of this new approach is to generate cost functions that represent the a priori knowledge necessary for phase unwrapping. Significant works have been proposed following Costantini's idea, based on network setup between selected points. In [9], for instance, a variation of Costantini's MCF method is presented, that avoids low coherence areas and uses instead isolated man made or natural point scattering objects to derive the deformation parameters of large scale land surface changes. Lanari et al. [10] investigate the phase retrieval capability of an improved MCF phase unwrapping approach where the procedure is implemented via the cascade of two steps, each based on the conventional MCF algorithm, that allow to exploit both the spatial characteristics and the temporal relationship among the produced interferograms.

In this work we present a new phase retrieval algorithm for InSAR interferograms. The algorithm is based on two steps: first, a mask based on the coherence map is applied to the interferogram, after phase unwrapping via the MCF approach, to remove phase values in the low coherence regions; second the reconstruction of the missing phase values is performed through

a Complex Ginzburg-Landau (CGL) inpainting scheme, as recently applied in [11].
This process works as a restoration technique of SAR interferograms by improving their quality and final quality of DEMs. The phase retrieval technique has been applied to ERS-1 and ERS-2 data sets and shows to work well if compared to the MCF approach [9], [12]. Its accuracy strongly depends on the amount of phase noise in each interferogram and on the resulting coherence among interferograms.

2. PHASE RETRIEVAL BY DIFFUSION AND INPAINTING

For the purpose of inpainting, we apply a high–contrast inpainting scheme based on the CGL equation. This equation was developed by Ginzburg and Landau to model phase transitions in superconductors near the critical temperature and has been recently applied to image restoration [11]. The null phase values are retrieved through a reaction-diffusion mechanism where the diffusion evolves mostly along edges and not across, thus avoiding smoothing effects.
We illustrate briefly the CGL inpainting scheme. Let D be the domain of the phase values and denote as $\Omega \subset D$ the inpainting domain where there are the phase values to be restored. In the CGL scheme, we solve in Ω the CGL equation

$$\Delta u + \frac{1}{\varepsilon^2}\left(1 - |u|^2\right) u = 0\,, \tag{1}$$

where $u : D \to \mathbb{C}$ is a complex valued function. Identify the function $\overline{u}$ as the real part of u, normalized and scaled, so that $\overline{u} : D \to [-1, 1]$. A complex valued solution u of the CGL equation has a unit modulus almost everywhere. Since the real part of the solution may contain any value in the interval $[-1, 1]$ and the boundary conditions for (1) are given by the phase values at the boundary of Ω, correspondingly the imaginary part is computed as

$$\Im(u) = \sqrt{1 - (\overline{u})^2} \tag{2}$$

such that $|u| = 1$. It can be shown that ε tunes the transition width between the phases ± 1 and thus the sharpness of the recostructed edges. Smaller ε results in sharper transition.
Hence the problem of inpainting consists in solving (1) with a suitably chosen parameter ε and the Dirichlet boundary conditions as described in [11]. For this purpose, a relaxation procedure is used and the associated reaction-diffusion equation (3), with arbitrary initial condition, is solved

$$\frac{\partial u}{\partial t} = \Delta u + \frac{1}{\varepsilon^2}\left(1 - |u|^2\right) u \tag{3}$$

up to a stationary point in time, i.e. until a give tolerance $\|\frac{\partial u}{\partial t}\| \leq \text{tol}$. The stationary point of (3) gives a solution of (1).

3. EXPERIMENTS AND RESULTS

We start from an unwrapped interferogram and its corresponding coherence map, as shown in Figure 1.
The algorithm is applied to a selected area of the interferogram; for this area, using the coherence map and fixing an appropriate threshold, a mask is built in order to define the inpainting domain as the area of the interferogram where the pixels have a value of correlation coefficient lower than the threshold. A new interferogram has been produced by removing the phase values with correlation coefficients less than the threshold. In Figure 2 the selected area of the interferogram, the coherence map and the new interferogram after the mask application are illustrated.
In the new interferogram the dark pixels underline the absence of phase values because of the mask application. At this point, the missing values have been restored by applying the inpainting scheme described in Section (2). Figure 3 shows final interferogram.

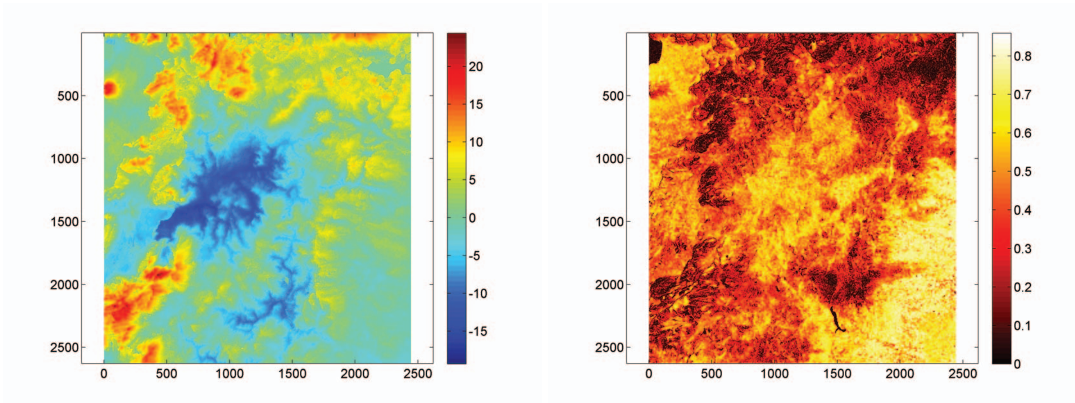

Fig. 1. Interferogram centered on Ariano Irpino (AV), Italy, produced through a couple of SAR images acquired with ERS-1 and ERS-2 on 13 and 14 of July 1995 (left) and its coherence map (right)

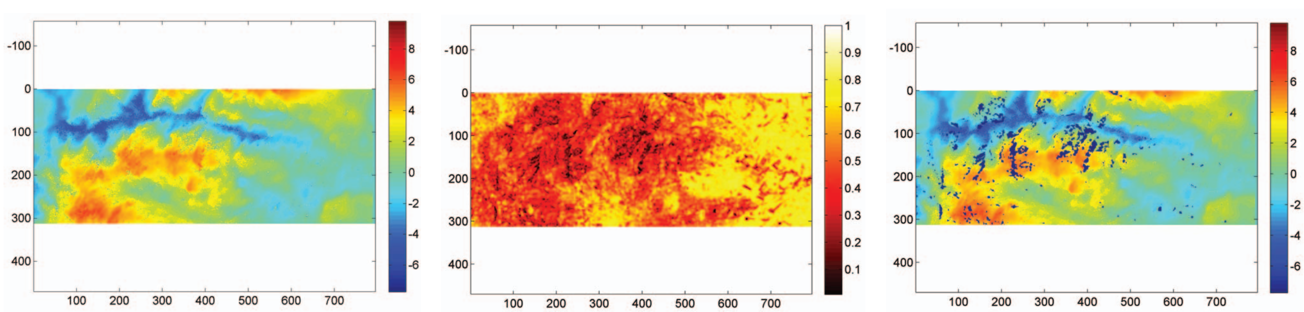

Fig. 2. In sequence: a selected area of interferogram of Figure 1 (left); its coherence map (center) and the new interferogram after mask application (right).

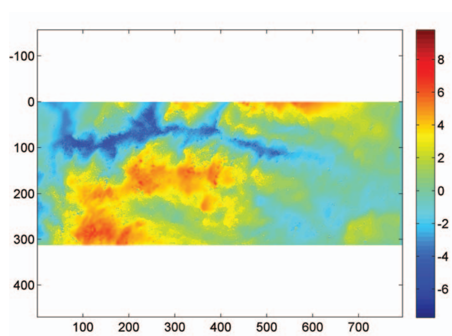

Fig. 3. Interferogram restored with the inpainting algorithm.

In order to get a measure of the improvement of the algorithm, it may be useful to calculate the Signal–to–Noise Ratio (SNR). The quantity

$$SNR = 20 \log \left(\frac{N_t}{N_r} \right) \tag{4}$$

gives an expression for the SNR by taking into account the number of total pixels N_t and the number of residuals N_r, where the residuals are those points around which a close integral of the phase difference gives a non–zero result (refer to [13] for more details). The SNR for the interferogram with inpainting is 27.98 dB versus 27.34 dB of SNR for the MCF interferogram. From a more thorough analysis, results also show that the proposed algorithm works better than MCF if the low coherence area has relatively small dimensions and has strong noise level whereas in the presence of low phase noise the MCF is performing better. One of the main points under investigation is the possibility to not discard the low–coherence phase values but to use them in the inpainting equation. Results seem to be very promising, especially in the medium–coherence area, where the preserved information acts as a driving function for inpainted phase values.

4. REFERENCES

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