DIRECT ESTIMATION OF FARADAY ROTATION AND OTHER SYSTEM DISTORTION PARAMETERS FROM POLARIMETRIC SAR DATA

Mariko Burgin[1], *Mahta Moghaddam*[1], *and Anthony Freeman*[2]

[1]Department of Electrical Engineering and Computer Science
University of Michigan, Ann Arbor, MI 48109-2122, USA
mburgin@umich.edu, mmoghadd@umich.edu

[2]Jet Propulsion Laboratory, California Institute of Technology
4800 Oak Grove Drive, Pasadena, CA 91109
tony.freeman@jpl.nasa.gov

1. INTRODUCTION

One of the major impacts of the Earth's ionosphere on spaceborne radar signals is the rotation of the polarization vector, also called the Faraday rotation (FR) effect. The FR effect increases with decreasing frequency. It is reported [1] that estimated maximum FR values under peak ionospheric total electron content (TEC) conditions range from 2.5 degrees (C-band) to more than 320 degrees (P-band). The FR effect depends on the electron density along the propagation path, the Earth's magnetic field, the radio frequency and the path direction of the signal, all of which vary in space and time. Low-frequency radars – radars operating at L-band and lower – must therefore be well calibrated not only for system distortion terms, but also for FR. Several recent techniques have been proposed for estimating FR and removing its effects from radar signals, though in most cases the range of applicability of the techniques has been small. In this paper we present a method for direct estimation of FR, along with several other system distortion parameters, which is applicable to a large range of Faraday rotation angles and can therefore be applied to low frequency radars. The solution is posed as a nonlinear optimization problem for the polarimetric radar system model. The applicability of the model is demonstrated through several examples and sensitivity studies.

2. STATEMENT OF THE PROBLEM

The fundamental problem in polarimetric radar calibration is to find the true scattering matrix S based on the measured scattering matrix M. The polarimetric radar system model is defined as follows [2]:

$$\begin{bmatrix} M_{HH} & M_{VH} \\ M_{HV} & M_{VV} \end{bmatrix} = A(r,\theta)e^{j\varphi}\begin{bmatrix} 1 & \delta_2 \\ \delta_1 & f_1 \end{bmatrix}\begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix}\begin{bmatrix} S_{HH} & S_{VH} \\ S_{HV} & S_{VV} \end{bmatrix}\begin{bmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{bmatrix}\begin{bmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{bmatrix}$$

The system model can be written in a compact notation as:

$$M = Ae^{j\varphi}R^{T}R_{F}SR_{F}T$$

In the above equation, S is the scattering matrix, R_F is the one-way Faraday rotation matrix, R is the receive distortion matrix (of the radar system) and T is the transmit distortion matrix (of the radar system). R and T contain the cross talk values δ_i (i = 1,2,3,4) and the channel amplitude and phase imbalance terms f_1 and f_2. $A(r,\theta)$ is a real factor representing the overall gain of the radar system and $e^{j\varphi}$ is a complex factor, which represents the round-trip phase delay and system dependent phase effects on a signal. The term $A(r,\theta)\ e^{j\varphi}$ and the matrix N, which represent the additive noise terms in each measurement, can be neglected because they can be easily mitigated or neutralized.

Many methods exist to calibrate radar systems by solving for the parameters of the foregoing system model. For example, in [3] it has been assumed that the Faraday rotation angle is zero $\Omega = 0°$), that $S_{HV} = S_{VH}$, that like and cross-polarized returns are uncorrelated, and that δ_1 and δ_2 are small compared to f_1 and f_2. The method requires the use of external targets for calibration. The analysis in [2] is based on the assumption that a SAR system can be built with small levels of crosstalk or that the crosstalk amplitude and phase are stable and measureable, and can therefore be removed ($\delta = 0°$). Again, $S_{HV} = S_{VH}$ is assumed, which allows the recovery of the relative phase between f_1 and f_2 and the ratio of the amplitudes between f_1 and f_2. This method also requires an external target to fully calibrate for f_1 and f_2. If all other distortions are presumed to have been removed, the method of Bickel & Bates [4] can then be applied to solve for Ω, the FR angle. More recently, an approach has been proposed for simultaneous system calibration and FR estimation [5], whereby the system model can be transformed into the form [3] with the help of the assumptions $\delta_3 = \delta_1$, $\delta_4 = \delta_2$, and the knowledge that the Faraday rotation and the cross talk values are known to be small. In this case, the Faraday rotation is still present, but nested in new parameters. This method can be successfully used to estimate small values of FR, up to about 10 degrees, but has not yet been demonstrated for larger Faraday rotation angles. In this paper, we describe a method for direct estimation of Faraday rotation effect, simultaneously with the other system distortion parameters, using only a few simple assumptions.

3. APPROACH

In this paper, we assume that the FR angle is known to within an uncertainty (up to +/- 10° of truth). This can be achieved by predicting the FR angle based on approximate distributions of the electron content in the ionosphere available from measurements and models such as the International Reference Ionosphere.

Further assumptions are a reciprocal crosstalk, i.e., identical crosstalk on transmit and receive ($\delta_3 = \delta_1$, $\delta_4 = \delta_2$), and backscatter reciprocity ($S_{HV} = S_{VH}$). The reference data are generated based on known (simulated) values for the FR angle, the crosstalk values and channel amplitude and phase imbalance terms. The reference data set is then available as 16 complex valued pairs of the form $M_{HH}M_{HH}^*$, corresponding to 32 "known" values.

The calibration procedure is divided into two major parts to ease optimization. The first part is solely concerned with the pre-estimation of the channel imbalance terms f_1 and f_2. The second part is solving the nonlinear set of equations to retrieve the FR angle Ω. In the first part, an initial guess for the FR angle, as well as a guess for f_1, f_2, δ_1, δ_2, δ_3 and δ_4 are needed to derive an accurate estimate for f_1 and f_2. The a priori estimates for system distortion parameters are generally available from ground testing and pre-flight system calibration. The optimization of the first part of the problem is achieved by sweeping the values of f_1 and f_2 and calculating the quantity $A = abs(Im\{S_{HV}S_{VH}^* + S_{VH}S_{HV}^*\})$ for each f_1, f_2 pair . The values for f_1 and f_2 can then be retrieved by minimizing A. Indeed, for a perfectly calibrated system, the value of A is zero. This quantity is successful to retrieve f_1 and f_2 and was found by defining and investigating a channel imbalance metric $1/2*abs(M_{HV} - M_{VH})^\wedge 2/(M_{HH}M_{HH}^* + M_{VV}M_{VV}^*)$ similar to [5]. In order to calculate A for each f_1, f_2 pair, the system model in [6] is used. It relates the measured scattering matrix M to the true scattering matrix S via nested parameters, which can be calculated based on the sweeped f_1 and f_2, and the guessed d_1, d_2 and FR angle Ω. The quantity A can be calculated by inverting this system model for the scattering matrix S and generating the required $S_{HH}S_{HH}^*$ terms.

In the second part, f_1 and f_2 can be assumed known, and the remaining unknowns, which include FR angle, δ_1 and δ_2, as well as the true scattering matrix, have to be found. Forming the like- and cross-products between the four terms in the measured scattering matrix M of the full system model and considering their real and imaginary part separately, we have a system of equations with 32 knowns and 20 unknowns. This nonlinear system of equations is solved by minimizing a cost function using methods based on the conjugate gradient method. Two investigated iterative methods for seeking the minimum of the cost function are the Secant and the Newton-Raphson method. Both require a set of equations that is twice differentiable. The Secant method generally shows the better performance, as well as a much faster execution time and simplicity. Also, the Fletcher-Reeves and Polak-Ribière formulas are compared; the Fletcher-Reeves formula gives in general more accurate results.

4. RESULTS AND CONCLUSION

The proposed approach is implemented and validated using actual radar data for several cases. The cases considered represent a large range of Faraday rotation angles as well as a variety of polarimetric radar scattering matrices. The spaceborne polarization-rotated scattering matrices are generated from airborne SAR (AIRSAR) data by simulating realistic FR values for the locations of the AIRSAR data acquisitions. It is found that if the guessed FR angle lies within 10° of the true FR angle, it is possible to retrieve the true FR to within 3°. Results so far indicate that the simultaneous retrieval of δ_1 and δ_2 is more difficult, as the current cost function is somewhat insensitive to these cross talk parameters. The range of applicability and sensitivity to various parameters is investigated and will be discussed at the talk. It has been shown that this optimization-based technique can estimate FR with high accuracy even for large values of the variable, up to 40 degrees. The uniqueness and stability of the solution are also investigated as the value of FR is increased, since increased FR makes the optimization problem progressively more nonlinear.

5. REFERENCES

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