# The edgelist algorithm for constraining phase unwrapped solutions with additional geodetic information

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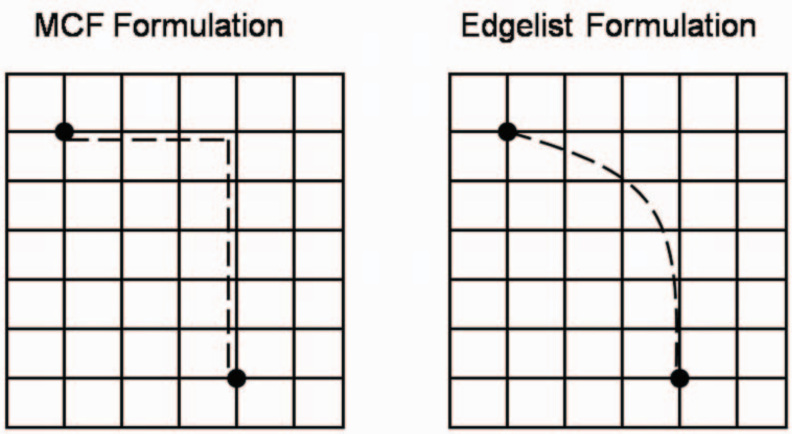
Phase unwrapping as used in InSAR geodesy is the reconstruction of absolute phase from measured phase known only modulo $2\pi$ on a finite grid of points in either 2 two or three dimensions. Many effective methods have been developed for unwrapping SAR interferograms [1-5], however phase unwrapping is a key step in many other coherent imaging techniques as well. Network programming approaches [3-4] have so far proven to be the most effective of these. The minimum cost flow (MCF) phase unwrapping algorithm developed by Costantini [3], describes a global cost minimization problem involving flow between phase residues computed over closed loops.

The recently developed "edgelist" algorithm [6-7], replaces the MCF formulation with its dual formulation where the closed loops are replaced by reliable edges as the basic construct. The edgelist formulation has several advantages over the original MCF formulation – it simplifies the representation of multi-dimensional phase unwrapping, it allows incorporation of data from external sources, like GPS, where available to better constrain the unwrapped solution, and it treats regularly sampled or sparsely sampled data alike. It thus is particularly applicable to time series InSAR where data are often irregularly spaced in time and individual interferograms can be corrupted with large decorrelated regions. Similar to the MCF network problem, our formulation also exhibits total unimodularity (TUM) which enables us to solve the integer program using efficient linear programming tools. The validity of the new formulation and the dual formulation approach has been verified by independent development of similar techniques by Costantini et al [8]. We have already illustrated the ability of this new unwrapping method to incorporate *a priori* information related to the geophysical phenomena and the area being studied, by applying it to a PS-InSAR dataset from the creeping section of the Central San Andreas Fault [6-7].

The biggest advantage of the edgelist formulation, however, is its ability to incorporate external geodetic measurements as constraints. The original MCF formulation is unable to incorporate such constraints due to the violation of total unimodularity property (See Fig 1). In case of the edgelist formulation, additional measurements can be easily included as constraints by addition of external edges to the unwrapping grid (See Fig 1). As the basic construct of the edgelist formulation is an edge, addition of extra edges does not violate the total unimodularity property of the formulation. This particular aspect of the formulation and its effectiveness in constraining solutions has not been addressed in any previous work [6-8].




$$\sum_{P}^{Q} K_{ij} = \Delta N_{gps} - \sum_{P}^{Q} \left[ \frac{\psi_i - \psi_j}{2\pi} \right]$$

$$n_P - n_Q = \Delta N_{gps} - \left[ \frac{\psi_P - \psi_Q}{2\pi} \right]$$

**Figure 1.** Image showing the incorporation of external GPS constraints in unwrapping of a regularly sampled SAR interferogram. The corresponding constraints for the MCF formulation [3] and the edgelist formulation [6] are also shown. Only in the case of the edgelist formulation is the constraint matrix totally unimodular and hence suitable for solving using linear programming solvers [6].

In this work, we apply the edgelist phase unwrapping algorithm to accurately extract elevation information from ALOS PALSAR interferograms in regions where traditional unwrapping techniques fail. We incrementally incorporate additional information from the SRTM digital elevation models [10], to improve the unwrapping solution and determine the effectiveness of these additional constraints in constraining the solution. The experiment also allows us to determine the ability of algorithm to use *a priori* information to unwrap across large decorrelated areas. We also attempt to understand the effect of these additional constraints on the cost functions of the edges in the immediate neighborhood of the constrained locations. Specifically, we present examples from mountainous and vegetated terrain where we expect conventional unwrapping techniques to fair poorly. We also evaluate the computational performance of our new algorithm, implemented using the CLP solver [10], against that of two popular phase unwrapping algorithms – MCF [3] and SNAPHU [4].

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