ESTIMATION OF VIRTUAL DIMENSIONALITY IN HYPERSPECTRAL IMAGERY BY LINEAR SPECTRAL MIXTURE ANALYSIS

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ABSTRACT

Virtual dimensionality (VD) was originally developed for estimating the number of spectrally distinct signatures present in hyperspectral data. The effectiveness of the VD is determined by the technique used for VD estimation. This paper develops an orthogonal subspace projection (OSP) technique to estimate the VD. The idea is derived from linear spectral mixture analysis. A similar idea was also previously investigated by the signal subspace estimate (SSE) and later improved by hyperspectral signal subspace identification by minimum error (HySime). Interestingly, with an appropriate interpretation the proposed OSP technique includes the SSE/HySime as its special case. In order to demonstrate its utility experiments using synthetic images and real image data sets are conducted for performance analysis.

1. INTRODUCTION

Due to high spectral resolution provided by a hyperspectral imaging sensor, a hyperspectral image can uncover many unknown spectral signal sources which cannot be identified a priori. Therefore, it is very challenging and difficult to determine how many spectral signal sources are present in a hyperspectral image. Recently, virtual dimensionality (VD) was defined as the number of spectrally distinct signatures in hyperspectral imagery to address this issue [1] where a technique developed by Harsanyi et al. [2], Harsany-Farrand-Chang (HFC) method was developed to estimate the VD. Since a spectrally distinct signature is determined by different applications such as endmember extraction, anomaly detection etc., the VD also varies with how a spectrally distinct signature is interpreted. While the HFC method is designed based on the observed data properties specified by signatures that can be discriminated spectrally band-by-band, this paper develops a new approach to VD estimation based on data representation in a certain form, specifically based on linear spectral mixture analysis (LSMA). With the LSMA in mind the proposed approach interprets a spectrally distinct signature as an image endmember that can be used to specify a particular spectral class. Accordingly, the VD is then interpreted as the number of image endmembers, p to be used to form a linear mixture for unmixing. In order to make this approach work, two issues, determining the value of p and finding a desired set of p image endmembers, need to be considered together as a single issue. This is quite different from the HFC method which does not require finding specific spectrally distinct signatures due to its use of the Neyman-Pearson detection theory where a binary hypothesis testing problem is formulated to test if a Neyman-Pearosn detector fails and the number of test failures is the VD.

By taking advantage of a recent technique, called Orthogonal Subspace Projection (OSP) develop for LSMA [3] an OSP-based method can be derived to estimate the VD where spectrally distinct signatures are defined as virtual endmembers (VEs) that are used for linear spectral unmixing where the mixed error is used as a criterion to determine the VD. A similar idea to the proposed OSP method was also investigated in [4], called signal subspace estimation (SSE) and later improved to be called hyperspectral signal subspace identification by minimum error (HySime) [5] based on minimization of the estimation error caused by signal subspaces spanned by spectral signal sources. Both the OSP and the SSE/HySime use a growing number of spectral signal sources to form signal subspaces via a linear mixing model to represent the data. The number of spectral signal sources that yields the minimal signal subspace estimate error is the desired VD. While both SSE/HySime and the proposed OSP method share the same concept of using signal subspaces to find best linear representation in some sense of optimality, i.e., minimum mean squared error for SSE/HySime and the unmixed error for the OSP method, there are also two prominent differences between these two approaches. One is that the SSE/HySime requires noise covariance estimation which is not needed in the OSP method. As a result, different noise estimation techniques will produce different values of the VD. To the contrary, the OSP-estimated VD varies with an algorithm used to find VEs. Consequently, different algorithms generally result in different values of the VD. This is not the case for SSE/HySime which uses the singular value decomposition (SVD) to generate signal sources and does not require a specific algorithm to be used. As a result, the SSE/HySime always produces a constant value of the VD regardless of applications which generally require various algorithms to be used for specific data processing. So, if the SVD is used in the OSP method to generate VEs, the SSE/HySime can be treated as its special case.

2. LINEAR SPECTRAL MIXING METHODS

The HFC method works well when the spectrally distinct signatures contribute little to data variances due to its small sample size in which case only sample mean accounts for its presence. This section considers a rather different approach. Instead of characterizing the spectral distinct signatures by statistics the proposed approach is to estimate the VD by seeking a set of spectral distinct signatures that can best represent the entire data in terms of a linear form. Such an idea can be traced back to linear spectral mixture analysis (LSMA) where a data sample vector is assumed to be represented by a linear mixture of a finite set of so-called endmembers. One early LSMA-based method was developed by SSE [4] and HySime [5]. This section presents an LSMA-based OSP method. Both methods are closely related in context of LSMA.

Suppose that there are p spectral signal sources, $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_p\}$ present in the data and every data sample vector $\mathbf{r}_i$ can be expressed by a linear mixture of these p spectral signal sources as follows

$$\mathbf{r}_i = \mathbf{S}_p \boldsymbol{\alpha}_i + \mathbf{n}_i \tag{1}$$

where $\mathbf{S}_p = [\mathbf{s}_1 \mathbf{s}_2 \cdots \mathbf{s}_p]$ is a signal matrix made up of the p spectral signal sources, $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_p\}$ and $\mathbf{n}_i$ can be interpreted as noise vector or model error vector. Let $\mathbf{P}_p = \mathbf{S}_p \left(\mathbf{S}_p^T \mathbf{S}_p\right)^{-1} \mathbf{S}_p^T$ be the p-signal projection matrix which maps $\mathbf{r}_i$ into the space spanned by the p spectral signal sources, $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_p\}$. From (1) the sample mean vector $\boldsymbol{\mu}$ can be expressed by

$$\boldsymbol{\mu} = (1/N)\left[\mathbf{S}_p \left(\sum_{i=1}^N \boldsymbol{\alpha}_i\right) + \sum_{i=1}^N \mathbf{n}_i\right] = \mathbf{S}_p \bar{\boldsymbol{\alpha}}_p + \bar{\mathbf{n}} \tag{2}$$

where $\bar{\boldsymbol{\alpha}}_p = (1/N)\sum_{i=1}^N \boldsymbol{\alpha}_i$ and $\bar{\mathbf{n}} = (1/N)\sum_{i=1}^N \mathbf{n}_i$ with the covariance matrix given by $\mathbf{R}_{\bar{\mathbf{n}}} = (1/N)\sum_{i=1}^N \mathbf{n}_i \mathbf{n}_i^T$. Using (2) we can obtain

$$E\left[(\mathbf{P}_p\boldsymbol{\mu})(\mathbf{P}_p\boldsymbol{\mu})^T\right] = \mathbf{S}_p\overline{\boldsymbol{\alpha}}_p\overline{\boldsymbol{\alpha}}_p^T\mathbf{S}_p^T + \mathbf{P}_p^T\mathbf{R}_{\overline{\mathbf{n}}}\mathbf{P}_p \tag{3}$$

By virtue of (3) we define

$$\mathrm{OSP}(p) = trace\left\{\mathbf{S}_p\overline{\boldsymbol{\alpha}}_p\overline{\boldsymbol{\alpha}}_p^T\mathbf{S}_p^T + \mathbf{P}_p^T\mathbf{R}_{\overline{\mathbf{n}}}\mathbf{P}_p\right\} = E\left[(\mathbf{P}_p\boldsymbol{\mu})^T(\mathbf{P}_p\boldsymbol{\mu})\right] \tag{4}$$

Theoretically, the value of OSP(p) in (4) increases as the value of p increases. For any given error threshold ε, VD can be determined by a stopping rule to OSP(p). The value of p determining the resulting OSP(p) is defined and denoted by $\mathrm{VD}^{\mathrm{OSP}}$. Two criteria are developed to detect the abrupt change of OSP(p) value. One which is based on the gradient, denoted by "∇", is defined by

$$\mathrm{VD}_{\mathrm{algorithm}}^{\mathrm{OSP},\nabla}(\varepsilon) = \arg\left\{\min_{1\le p\le L}\left|\frac{\mathrm{OSP}(p+1)}{\mathrm{OSP}(p)} - \frac{\mathrm{OSP}(p)}{\mathrm{OSP}(p-1)}\right| < \varepsilon\right\} \tag{5}$$

and the other which is based on the difference, denoted by minus "-", is defined by

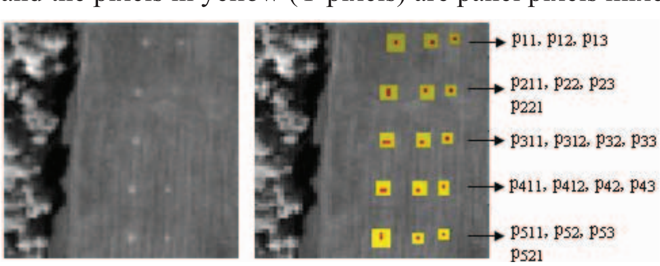
$$\mathrm{VD}_{\mathrm{algorithm}}^{\mathrm{OSP},-}(\varepsilon) = \arg\left\{\min_{1\le p\le L}\left|\mathrm{OSP}(p+1) - \mathrm{OSP}(p)\right| < \varepsilon\right\}. \tag{6}$$

where the sample mean vector $\boldsymbol{\mu}$ is normalized before projection so that the values of thresholds ε's for these two criteria are comparable for analysis. The threshold ε in (5) and (6) is generally selected according to a sudden drop or a clear gap between two consecutive p's in plots of the gradient in (5) and difference (6) versus the value of p.

It should be noted that (4), (5) or (6) actually involves two key parameters needed to be addressed. One is the error threshold ε which is already included in (5) and (6). The other parameter is the algorithm to be used to produce the p-signal matrix $\mathbf{S}_p$, which is not particularly specified in (5) and (6) but rather the term of "algorithm" is used in (5) and (6) for a generic expression. Since the OSP method is derived from the linear mixing approach, the spectral signal sources $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_p\}$ in (1) determined by $\mathrm{VD}^{\mathrm{OSP}}$ is referred to as "virtual endmembers" instead of image endmembers commonly used in linear spectral unmixing.

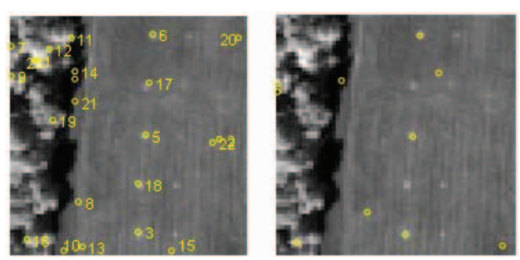
3. EXPERIMENTS

For performance evaluation and comparative analysis the image scene shown in Fig. 1 was used for experiments. It has size of 64×64 pixel vectors shown in Fig. 1(a) along with its ground truth provided in Fig. 1(b) which shows the precise spatial locations of these 19 panel pixels where red pixels (R pixels) are the panel center pixels and the pixels in yellow (Y pixels) are panel pixels mixed with the background.



(a) (b)

Figure 1. (a) A HYDICE panel scene which contains 15 panels; (b) Ground truth map of spatial locations of the 15 panels



(a) 23 VEs ATGP (b) 10 VEs by N-FINDR

Figure 2. VEs extracted by ATGP and N-FINDR

TABLE I. VD ESTIMATED FOR REAL IMAGE BY HFC AND NWHFC

P_F	10^{-1}	10^{-2}	10^{-3}	10^{-4}
HFC	14	11	9	9
NWHFC	20	14	13	13

TABLE II. VD ESTIMATED FOR REAL IMAGES BY OSP

	OSP-ATGP	OSP-NFINDR
gradient	23 (-70)	10 (-50)
diff	11 (-70)	10 (-50)

The VD estimated for the HYDICE scene in Fig. 1(a) by SSE and HySime is 10 and 20 respectively compared to the VD estimated by HFC/NWHFC Table I. Table II tabulates the VD estimated by OSP using ATGP and N-FINDR.

Fig. 2 shows VEs extracted by N-FINDR [6] and ATGP [7] according to the VD in Table II where the OSP-ATGP was the best which extracted all the five panel signatures in the scene while the N-FINDR could only extract 3 panel signatures. This experiment further demonstrated a key feature in VD estimation which is the algorithm used to extract VEs. If an algorithm is not used appropriately, the results will not be expected even the VD is estimated correctly.

4. CONCLUSIONS

This paper presents a new application of the LSMA in VD estimation. In context of LSMA the VD can be interpreted as the minimal number of signatures that best represent the data sample vectors in a linear mixing form. Two methods, SSE/HySime and OSP are investigated to materialize this idea. While both techniques make an attempt to find a signal subspace with minimum dimensionality to linearly represent data sample vectors with minimum linear mixing error, they also differ from each other in terms of design rationale. The SSE/HySime makes Gaussian noise assumption so that noise sample covariance matrix can be estimated by various noise estimation techniques. As a result, a different noise estimation technique may result in a different value of the VD. Second, the SSE/HySime produces a single value of the VD which independent of algorithms. Third, the SSE/HySime does not provide a means of finding the signal sources once the VD is determined. Although the SSE/HySime uses the singular value decomposition (SVD) to generate signal sources specified by singular vectors, these singular vectors are not necessarily to represent the desired signal sources. The OSP methods were developed to address these three issues. They do not need to estimate noise sample covariance matrix. So, no assumption of Gaussian noise is made. Furthermore, the value of the VD estimated by the OSP methods is also determined by two parameters, an error threshold ε which can be tuned by signal sources of interest and an algorithm which can be custom-designed to generate the signal sources for a particular application. By specifying the ε and the algorithm to be used to generate signal sources the OSP methods eventually remedy the second and third issues encountered in the SSE/HySime. Experimental results also demonstrate that the OSP methods are more flexible than the SSE/HySime in real practical applications.

5. REFERENCES

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