A STUDY ON EFFECTIVE DIELECTRIC CONSTANTS OF NON-SPHERICAL SNOWFLAKE AND MELTING HYDROMETEORS

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1. INTRODUCTION

The bright band, a layer of enhanced radar echo associated with melting hydrometeors, is often observed in stratiform rain. The microphysical properties of melting hydrometeors and their scattering and propagation effects have long been studied not only because of their importance in accurately estimating parameters of the precipitation from spaceborne radar and radiometers but also as a result of their negative influence on earth-satellite communication systems caused by attenuation and depolarization of radio signals. To characterize the properties of the melting layer a number of observations of the radar bright band have been made by both ground-based weather radars and multi-frequency airborne radars. However, one of the impediments to the study of the radar signature of the melting layer is the determination of effective dielectric constants of melting hydrometeors. Although a number of mixing formulas are available to compute the effective dielectric constants, their results vary to a great extent when water is involved in the mixture, such as in the case of melting snow[1][2]. It is also physically unclear as to how to select among these various formulas. Furthermore, questions remain as to whether these mixing formulas can be applied to computations of radar polarimetric parameters from non-spherical melting particles.

Recently, several approaches using numerical methods have been developed to derive the effective dielectric constants of melting hydrometeors, i.e., mixtures consisting of air, ice and water, based on more realistic melting models of particles, in which the composition of the melting hydrometeor is divided into a number of identical cells. Each of these cells is then assigned in a probabilistic way to be water, ice or air according to the distribution of fractional water contents for a particular particle. While the derived effective dielectric constants have been extensively tested at various wavelengths over a range of particle sizes, these numerical experiments have been restricted to the co-polarized scattering parameters from spherical particles[3][4][5]. As polarimetric radar has been increasingly used in the study of microphysical properties of hydrometeors, it will certainly provide additional information on melting processes. To account for polarimetric radar measurements from melting hydrometeors, it is necessary to move away from the restriction that the melting particles are spherical.

In this study, our primary focus is on the derivation of the effective dielectric constants of non-spherical particles that are mixtures of ice and water. The computational model for the ice-water particle is described by a collection of 128x128x128 cubic cells of identical size. Because of the use of such a high-resolution model, the particles can be described accurately not only with regard to shape but with respect to structure as well. The Cartesian components of the mean internal electric field of the particle, which are used to infer the effective dielectric constants, are calculated at each cell by the use of the Conjugate Gradient-Fast Fourier Transform (CGFFT) numerical method. In this work, we first check the validity of derived effective dielectric constant from non-spherical mixing particle by comparing the scattering and polarimetric parameters of a ice-water mixed spheroid obtained from the CGFFT to those computed from the T-matrix for a homogeneous particle with the same geometry as that of the mixed phase particle (such as size, shape and orientation) with an effective dielectric constant derived from the internal field of the mixed-phase particle. The accuracy of the effective dielectric constant can be judged by whether the scattering parameters of interest can accurately reproduce those of the exact solution, i.e., the T-matrix results. To extend our study to more complex yet realistic snowflakes we use the mesoscopic model [6], which is based on diffusion of vapor, anisotropic attachment and a boundary layer, to generate snow aggregates. From these results, we will be able to examine the differences in the scattering properties of the simulated aggregates and one or more of the simplified particle models, assuming that the particle mass is conserved. Among the most common shapes of snowflake models in radar applications are the sphere and spheroid with the aspect ratio in the later case determined by minimum and maximum dimensions of snow aggregates. These comparisons will enable us to check the validity and examine the accuracy of sphere/spheroid models in simulating radar-bright signatures.

The purpose of the effective dielectric constant is to reduce the complexity of the scattering calculations in the sense that this quantity, once obtained, may be applicable to a range of particle sizes, shapes and orientations. Having computed the effective dielectric constant for a particle with a specific shape, size, and orientation, a check is performed to see if the result, obtained from one realization (with a fixed size, shape and orientation), can be used to characterize a class of particle types (with arbitrary sizes, shapes and orientations) if the fractional ice-water contents of melting particles remain the same. Among the scattering and polarimatric parameters used for examination of effective dielectric constant in this study are: radar backscattering, extinction and scattering coefficients, asymmetry factor, differential reflectivity factor (ZDR), phase shift and linear polarization ratio (LDR). The ultimate objective is to examine whether the effective dielectric constant approach provides a means to compute radar and radiometric polarimetric scattering parameters from the melting layer in a relatively simple and accurate way.

2. APPROACHES AND RESULTS

Let **E**(r,λ) and **D**(r,λ) be the local electric and dielectric displacement fields within a composite material at location r at free-space wavelength λ, satisfying

$$\boldsymbol{D}(\boldsymbol{r},\lambda) = \varepsilon(\boldsymbol{r},\lambda)\boldsymbol{E}(\boldsymbol{r},\lambda), \tag{1}$$

where ε is the dielectric constant. In view of the local constitutive law described by the above equation, the bulk effective dielectric constant, ε_{eff}, is defined as the ratio of the volume averages of **D** and **E** fields (Stroud and Pan 1978)

$$\varepsilon_{eff} \iiint_V \boldsymbol{E}(\boldsymbol{r},\lambda)\boldsymbol{dv} = \iiint_V \boldsymbol{D}(\boldsymbol{r},\lambda)\boldsymbol{dv}\,. \tag{2}$$

If the particle, composed of two materials ε_1 and ε_2, is approximated by N small equal-volume elements, then the ε_{eff} can be written as

$$\varepsilon_{eff} = \frac{\varepsilon_1 \sum_{j\in M_1} \boldsymbol{E}_j + \varepsilon_2 \sum_{j\in M_2} \boldsymbol{E}_j}{\sum_{j\in M_1} \boldsymbol{E}_j + \sum_{j\in M_2} \boldsymbol{E}_j}\,. \tag{3}$$

The notations $\sum_{j\in M_1}$ and $\sum_{j\in M2}$ denote summations overall volume elements comprising materials 1 and 2, respectively. In this study, the internal fields appearing on the right-hand sides of (3) are computed by the CGFFT numerical method in which the volume enclosing the total particle is divided into 128×128×128 identical cells. The particle representation, described by the number of cells, is expected to be improved greatly as the computation will be performed on the NASA supercomputer system that should enable the study of more complex particle shapes.

2.1. Spheroidal Model

Fig. 1 shows an example of the comparisons made at Ka band (35 GHz) between the direct (CGFFT) computations from particle realizations and the T-matrix computations using the effective dielectric constant. For

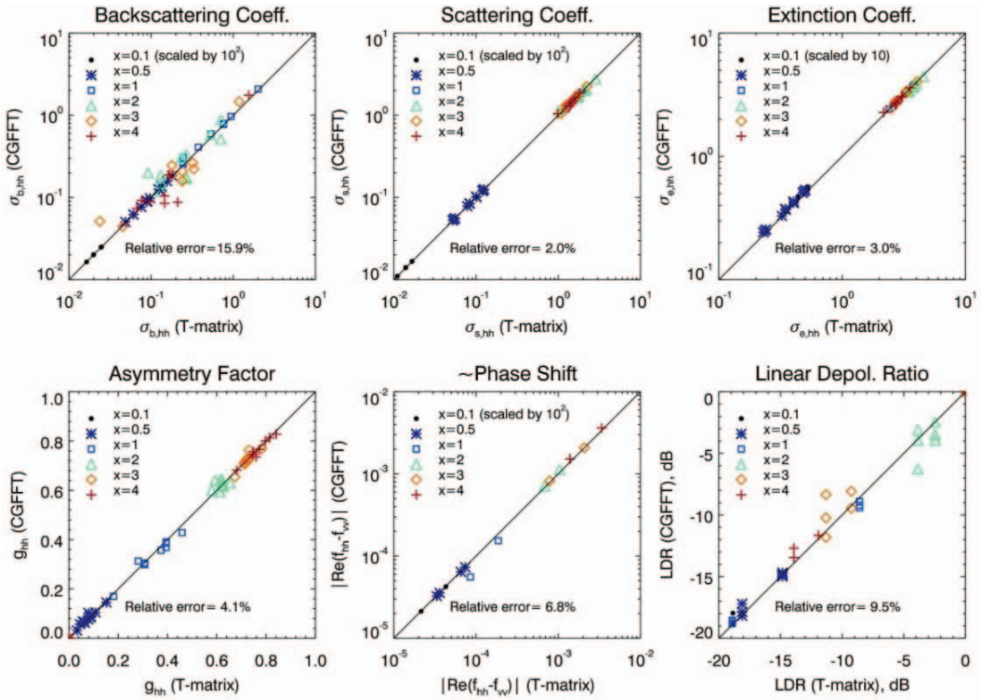

Fig.1 Comparisons of scattering parameters computed from the CGFFT direct computations and T-matrix for uniform particles as their effective dielectric constants are derived from particle realizations. Among these results are those from spheres and oblate and prolate spheroids. The scattering parameters involved in these comparisons are the backscattering coefficient (σ_b), scattering coefficient (σ_s), extinction coefficient (σ_e), asymmetry factor (g), absolute value of real part of difference of scattering amplitudes between horizontal and vertical polarizations ($|Re(f_{hh}-f_{vv})|$) (which is proportional to the phase shift of propagation between horizontal and vertical polarizations), and linear depolarization ratio (LDR). The subscripts, hh and vv, denote the copolarized returns for horizontal and vertical polarizations, respectively.

(a) (b) (c) (d)

Fig.2 Realizations of two perpendicular cross sections (top and bottom) of oblate (a and b) and prolate (c and d) spheroids used for the CGFFT computations as the particles are described by a collection of 128×128×128 identical cubic cells. Light and dark areas correspond to ice and water, respectively.

these computations the particle size parameter x, defined as 2πa/λ (where a is particle radius of equivolume and λ is wavelength), ranges from 0.1 to 4, where the water fraction is fixed at 0.3. For each size parameter the computations are carried out for oblate and prolate spheroids with axial ratios of 0.125 and 0.5 for oblates and 2 and 8 for prolates, as shown in Fig.2. It is evident that the CGFFT direct computations are in good agreement with the T-matrix despite some discrepancies that occur at large particle sizes (x=4) as a result of fluctuations in the scattering parameters from realization to realization. These results suggest the validity of effective dielectric constant of mixed-phase spheroids to compute co- and cross-polarization scattering parameters.

2.2. Crystal Growth Model

An investigation will be made into the effective dielectric constant and scattering properties of snow aggregates produced by the ice crystal growth model developed by Gravner and Griffeath [6]. The model-generated snowflakes are able to replicate most of the observed snow-crystal morphology. They, however, appear in highly complicated shapes and structures that require a fairly large amount of pixels (or cells) in order to compute their scattering parameters and effective dielectric constants. To fulfill this study, the CGFFT computation will be carried out by using the NASA supercomputer system (http://www.nccs.nasa.gov). Fig.3a displays an example of snow aggregates made of a collection of hexagonal columns with intruded ends of various sizes. The component particles, as shown in Fig.3b, are numerically "grown" following Gravner and Griffeath model, which is capable of generating realistic ice particles with the exquisite features observed in nature. Fig.3c provides an IDL-rendered view of snow aggregates composed of pristine dentritic crystals. By comparing the scattering results of modeled snow aggregates to those from spherical and spheroidal particles with the same masses, the accuracy of the sphere/spheroid model with the use of an effective dielectric constant can be tested.

3. REFERENCES

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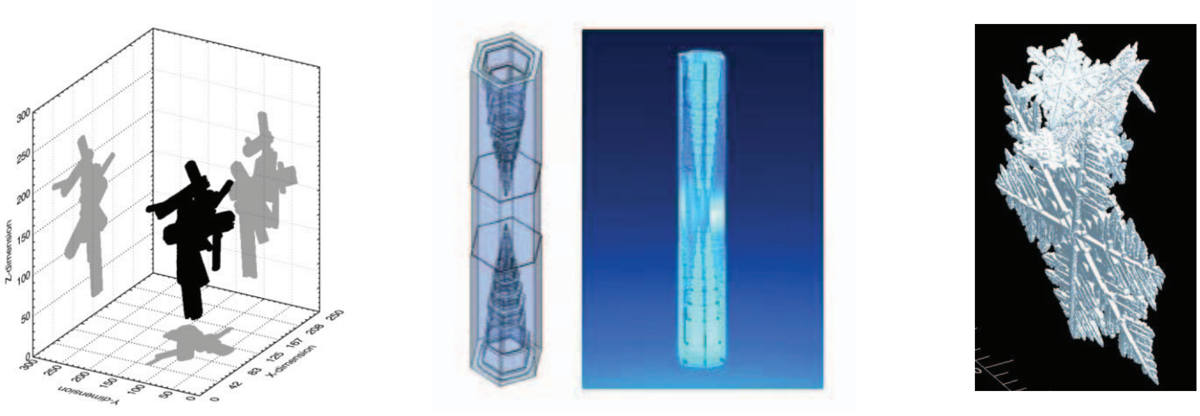
Bio: Liang Liao received the Ph.D. degree in Meteorology from the University of Utah in 1993. His research interests include the radio wave propagation, electric field scattering, and atmospheric radar and radiometer remote sensing on clouds and precipitation.



(a) (b) (c)

Fig.3 (a) Model-generated snow aggregates, (b) hexagonal hollow column and (c) IDL-rendered view of snow aggregates.