MODELING LIDAR SCENE SPARSITY USING COMPRESSIVE SENSING

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ABSTRACT

One of the major problems associated with LIDAR sensing is that significant amounts of data must be collected to obtain detailed topographical information about a region. Current efforts to solve this problem have focused on designing compression algorithms which operate on the collected data. Typical compression algorithms, however, require the collection of large amounts of data only to discard most of it in some transformed domain. Instead, the theory of compressive sensing has demonstrated that highly accurate signal reconstructions are achievable even when sampling below the Nyquist rate. Such sensing is clearly desirable for LIDAR range data compression if it can be achieved. One notes, however, that compressive sensing requires *a priori* knowledge of the sparsifying basis of the signal which is a major problem for LIDAR since that basis depends not only on the underlying scene complexity but also on the laser spot size and the distance to the target. For these reasons, the major goal of this research is to take the first steps in establishing a relationship between typical LIDAR scenes of varying complexity and the sparsity of the scene compressively sampled.

Index Terms— Compressive sensing, LIDAR, scene complexity, sparsity.

1. INTRODUCTION

Typical LIDAR data contains large amounts of information that impose significant problems in the storing, processing and transmitting tasks. While a number of LIDAR compression approaches have already been developed and discussed in the literature [2], [6], these assume uniform spatial scanning during acquisition, something that rarely occurs in practical applications. The theory of compressive sensing in which a signal is sampled with respect to a random basis offers the potential of overcoming this problem. Sampling a signal of length n using compressive sensing consists of sampling just a few random samples ($m << n$) using the inner product of the sampling function φ_k

$$y_k = \langle f, \varphi_k \rangle, \quad k = 1,..., m \qquad (1)$$

Here, y_k represents the k^{th} signal sample and the $\langle\,\rangle$ operator denotes the inner product. The attractiveness of compressive sensing comes from the fact that the signal f can be fully recovered with high probability under the conditions that the signal is sparse in some representation basis Ψ and sampled in some incoherent basis Φ. The sparsity condition is based on the concept that many natural signals can be represented without much perceptual loss by keeping the largest non-zero coefficients of the signal expressed in the proper orthonormal basis (e.g., Fourier, wavelets). In other words, when $\|f - f_s\|$ is small, with $f = \Psi x$, $f_s = \Psi x_s$ and x and x_s being the vector of full and sparse transformed coefficients, respectively. The incoherence condition is also important to compressive sensing because less coherent pairs yield precise reconstructions with fewer m measurements. In general, coherence measures the largest correlation between any two elements of the sensing basis Φ and the representation basis Ψ. This measure of coherence is given by Candés and Wakin [1]

$$\mu(\Phi, \Psi) = \sqrt{n} \cdot \max_{1 \le k, j \le n} \left|\langle \varphi_k, \psi_j \rangle\right| \qquad (2)$$

Large incoherence can be achieved with high probability given that a random orthonormal basis is selected as the measurement basis. Typically, random matrices are largely incoherent (on the order of $\sqrt{2\log n}$) with any fixed representation basis Ψ [1]. In addition to the sparsity and incoherence conditions, the number of m samples required for exact signal recovery of f is

$$m \geq C \cdot \mu^2(\Phi, \Psi) \cdot S \cdot \log n \qquad (3)$$

for some positive constant C and S number of non-zero coefficients of the signal represented in the Ψ basis. Finally, the problem of reconstruction using compressive sensing can be formulated as a convex program

$$\min_{\tilde{x} \in \mathbb{R}^n} ||\tilde{x}||_{l_1} \text{subject to } y_k = \langle \varphi_k, \Psi\tilde{x} \rangle, \forall \in M \qquad (4)$$

where the $||\ ||_{l_1}$ operator denotes the l_1 norm and the $\tilde{x}$ are the estimated coefficients of the signal expressed in the Ψ basis. The advantage of using the l_1 norm in equation 4 as the minimization problem is that it promotes sparsity while being more computationally tractable than the theoretically desirable l_0 norm.

One of the major issues in the application of compressive sensing to the problem of LIDAR range data compression is that a general basis of sparsity is not known *a priori*. Instead, it depends heavily on the laser pulse spot size, the sampling pattern, and the scene complexity. Thus, we are motivated in this paper to develop a model of the target scene within a compressive sensing framework in order to characterize the degree of sparsity and to determine the number of measurements required to achieve acceptable reconstruction quality.

2. APPROACH

In this research, a LIDAR system that uses random single-point data collection of randomly generated surfaces with distinct complexity was simulated in Matlab. Note that this is equivalent to using random impulsive sensing basis. Random surfaces were generated by a fractal-based iterative algorithm which uses a midpoint displacement in two dimensions to create corner points delimiting smaller but geometrically equal shapes (e.g., square facets). The space in between the corner points of each corresponding facet is filled using bilinear interpolation [3]. A total of seven surfaces of increasing complexity (i.e., facet sizes) which vary between 1 and 7 were generated. For the laser pulse simulation we assumed noiseless sampling and we set the laser footprint size equal to the size of the smallest facet in the surface of highest complexity. The surface was randomly sampled with single pulses; lossless transmission and Lambertian-scattering upon reflection were further assumed. Thus, each of the acquisitions was obtained using a set of m random measurements defined by the sampling functions φ_k. The set of m-random measurements at which surface reconstructions were obtained was m = {4, 40, 400, 4000, 8000, 16000}. The reconstruction of the surface from an incomplete set of measurements using compressive sensing was achieved with the l_1 minimization program described in eq. (4) using additional constraints. Estimates of the surface sparsity were computed by establishing a relationship between the number of m-measurements required and the lowest achievable MSE using the available set of reconstructions.

A total of 1000 reconstructions were obtained for each of the m-measurement and generated surfaces sets. Combining these, results in a total of 42,000 reconstructions used for the estimation of the corresponding MSEs for each m-measurement and surface complexity. The reconstruction algorithm used is the total variation (TV) obtained from the l_1-magic Matlab collection of subroutines developed by Candés and Romberg in 2005 [5].

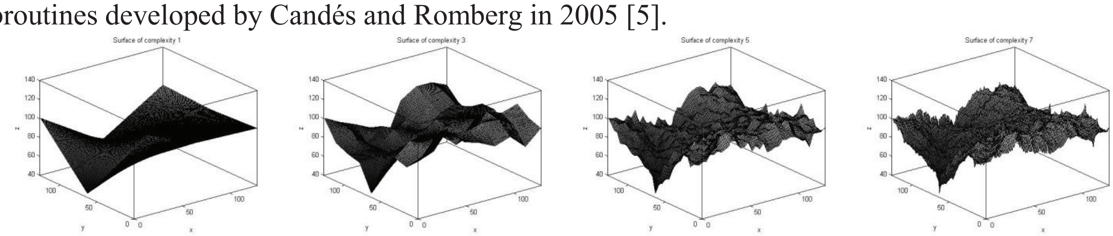

Figure 1: Generated surfaces a) Complexity 1, b) Complexity 3, c) Complexity 5, d) Complexity 7.

3. RESULTS

Each of the seven synthetic surfaces of size 129 × 129 was generated with a 100 meter mean level, a roughness of 30, and surface complexities ranging from 1 to 7. Examples of various surfaces are shown in figure 1. In general, the generated surfaces have a similar shape but distinct levels of complexity. Surfaces of lower complexity contain just a few big sized facets which compose the total surface area while surfaces of higher complexity contain a large number of small sized facets, introducing more detail.

An example of a compressive sensing reconstruction using m = 4000 random measurements of the generated surfaces is shown in figure 2 for a surface of complexity 4.

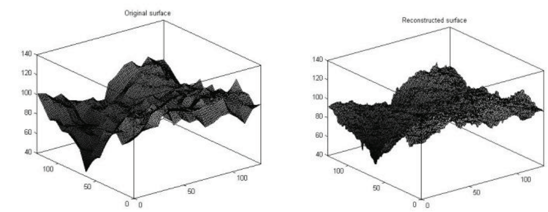

Figure 2: Surface reconstruction a) Original surface, b) Reconstructed surface

We note from figure 2 that the shape of the reconstructed signal resembles that of the original. The algorithm was not capable of recovering sharp edges formed between adjacent facets, however. The resulting mean squared error (MSE) of this reconstruction is of 5.713 with an approximate compression ratio of 4:1. To illustrate the resulting MSEs of all the reconstructions, the mean MSE is computed over the 1000 computed reconstructions for each value of m and each surface complexity. Values of m equal to 4, 40, 400, 4000, 8000, and 16000 were used. The results are plotted in figure 3 which also includes plots of the mean MSEs of surfaces with distinct complexity. Note that the MSEs of the surface complexities are very close to one another for all sets of m random measurements, excepting that of the lowest complexity surface.

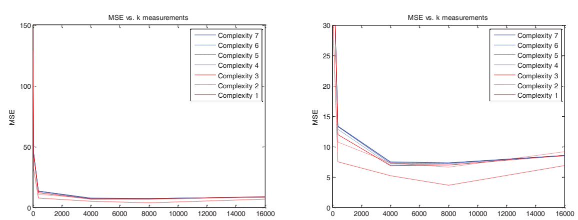

Figure 3: Reconstruction MSE as a function of k measurements. a) Mean MSE across reconstructions b) Zoomed plot

4. DISCUSSION

The results show that accurate reconstructions of the generated surfaces can be obtained using compressive sensing. Furthermore, figure 3 shows that the number of measurements required to obtain small MSEs appears to increase as the complexity of the surfaces increases. To establish this relationship more clearly, the Tukey statistical test described in [4] is implemented, evaluating the equality of pairs of mean MSEs. For each of the surface complexities, the mean MSE for each m is compared with the mean MSE corresponding to m = 16000. The minimum number of measurements for which the mean MSE for a given surface complexity is statistically equal to that for 16000 measurements is then selected as the minimum m satisfying equation 3. The resulting m*'s* corresponding to each of the surface complexities is given in table 1.

Table 1: Single return surface characterizations

Surface Complexity	Minimum m	Sparsity ratio $\frac{S_x}{S_7}$
1	1232	0.375
2	1924	0.586
3	2769	0.844
4	2919	0.889
5	3127	0.953
6	3267	0.995
7	3282	1

The third column of this table expresses a sparsity ratio relative to the surface of highest complexity. This relationship was obtained using equation 3 to construct the ratio given by

$$\frac{m_x}{m_7} \geq \frac{C \cdot \mu^2(\Phi,\Psi) \cdot S_x \cdot \log n}{C \cdot \mu^2(\Phi,\Psi) \cdot S_7 \cdot \log n} \qquad (5)$$

Simplification of this ratio can be achieved by eliminating the $\log n$ term because the generated surfaces are all of the same size as well as the constant C. Cancelation of the incoherence term is achieved by assuming that the complexity of the surface introduces no changes to the sparsifying basis which in this case is true. In general, we found with only a few exceptions that increasing surface complexity imposes higher m-measurement requirements and decreasing complexity reduces the number of m-measurements necessary for accurate reconstructions. Thus, if we can somehow estimate the surface complexity, then we can potentially use this to estimate the number of randomly-distributed LIDAR pulses that we need to bounce off that surface in order to accurately reconstruct it.

5. CONCLUSIONS

In this paper we explored the correlation between LIDAR surfaces of distinct complexity and their sparsity to establish a complexity-sparsity relationship. In general it was found that the number of measurements required for accurate surface reconstruction increases as the complexity of the surface increases (i.e., more and smaller facets). We note also that the relative sparsity ratios are close to one when one surface is similar in complexity to another which might be advantageous in finding the sparsifying basis for the LIDAR range data.

6. REFERENCES

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