ALGAE: A FAST ALGEBRAIC ESTIMATION OF INTERFEROGRAM PHASE OFFSETS IN SPACE VARYING GEOMETRIES

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1. ABSTRACT

One of the most useful applications of Interferometric Synthetic Aperture Radar (InSAR) systems is the estimation of Digital Elevation Models (DEMs) of the earth's surface. It is well known that the flattened, unwrapped phase φ related to a target p and an interferogram i is ideally proportional to its height, h [1]. Nonetheless, interferometric phases are affected by slow-varying random oscillations, due mainly to the propagation through the atmosphere, wrong sensor position estimation and deformations. Since these phase offsets are different for each interferogram, unless of taking account of this effect, a direct terrain elevations retrieval is impossible. Without neither ground reference points nor a model that accounts for the physics of propagation disturbances the problem can be tackled by compensating for a constant offset, α_i, in each interferogram, under the hypothesis of spatially smooth behavior of propagation disturbances. In formula:

$$\varphi_i^p = K_i^p h^p + \alpha_i \tag{1}$$

where K is the height-to-phase conversion factor. The most common and simple solution of this problem is to lock the phase of each interferogram at a stable reference point and estimating the ground heights with respect to it. This kind of solution results from the fact that, for the spaceborne case, the spatial variation of K can be neglected, at least on limited areas, resulting in the phase differences to be proportional to the differences of altitude:

$$\Delta\varphi_i = K_i \Delta h \tag{2}$$

However, space varying geometries introduce a significant change with respect to the previous case. A typical example is an airborne SAR system acquisition, in which the incidence angle sensitivity undergoes a dramatic increase with respect to the spaceborne case, due to the closer distance between the Radar sensor and the target. This leads to a high spatial variability of the normal baselines, resulting in the phase differences between two nearby points to be also dependent on the derivative of K times the ground elevation. Accordingly, Eq. 2 becomes:

$$\Delta\varphi_i = K_i \Delta h + \Delta K_i h \tag{3}$$

Consequently, the same value of interferometric phase implies a different height for a different location of the target and the phase locking effect will change according to the chosen reference point. This behavior affects dramatically the DEM estimation through Eq. 1, since the phase contributions resulting from the additional term will be mixed up between elevation errors and residual components (deformation, atmosphere, uncompensated sensor motion). The dependence with the reference point, which can usually be neglected in the spaceborne case, prevents the employment of the phase locking approach. Therefore, direct application of standard satellite techniques like Permanent Scatterers Interferometry (PSI) , [2], is hindered. However, terrain topography can still be retrieved by solving directly the linear system composed by Eq. (1) for $i = 1...N_I$ and $p = 1...N_P$.

From the analysis of the linear system above cited, the existence of a null space can be defined upon the condition that the height-to-phase conversion factors are characterized on all interferograms by the same spatial variation along the slant range, azimuth coordinates, up to a scale factor. Depending on the shape of the null space heights, 2 typical cases can be distinguished:

1. if the K have a constant spatial distribution the null space heights coincide to a constant offset and terrain heights can be retrieved with respect to them (spaceborne systems case over limited areas);
2. if the K are distributed with the same spatial variation along the interferograms, the null space height is proportional for each point at the inverse of its height-to-phase conversion factor (airborne systems case or spaceborne systems over extended areas)

Finally, the null space does not exist if the K are distributed with different spatial variations on different interferograms. This is the case of airborne systems with sensor motion corrections or spaceborne case with orbital corrections. Under this hypothesis, the problem is well posed. Nonetheless, the linear system is ill-conditioned and the retrieval of absolute ground heights hindered. In order to properly cope with this issue we propose an algebraic approach for the interferogram phase offsets estimation that takes into account in a simple way of the spatial variation of the normal baselines. The proposed approach retrieves terrain topography by allowing a 1-dimensional null space, ensuring the robustness of the solution. Since the effect of the imposed null space can not be described as a simple high offset, but rather as a slowly deformation along the slant range direction, to fix this artefact the null space component, opportunely weighted, is reintroduced in the solution in order to give the best match with the reference DEM. This approach will be hereinafter referred to as Algebraic Altitude Estimation (ALGAE). This method allows a fast retrieval of phase offsets and topography by means of algebraic estimation (best least square solution). The solution is consistent with the phase data and takes account of the spatial variability of K. Certainly, the same problem can be faced from a statistical point of view. For example, in literature the problem has been studied in [3], in which interferogram phase-offsets are obtained by means of a Maximum Likelihood estimation taking into accounts a spaceborne system case.

Experimental results have been obtained basing on a data-set of multi-polarimetric and multi-baseline SAR images, at P band, acquired by DLR's E-SAR on the Krycklan catchment, in northern Sweden, in the framework of the ESA BIOSAR 2008 campaign. Since the selected data-set has been acquired in a forest scenario, interferometric ground phases have to be estimated. This task has been accomplished according to [4], by exploiting the concepts of Algebraic Synthesis [5] and Phase Linking [6]. Firstly, the AS technique is employed for the retrieval of ground only contributions within the data, basing on the availability of multi-polarimetric information. Then, the Phase Linking algorithm is exploited in order to retrieve the best estimate of the ground phases with respect to a common Master image. Finally, the retrieved ground phases have been unwrapped, in such a way as to cast the problem of DEM estimation in the form depicted by eq. (1). The top left panel of Fig. (1) shows the error between the LIDAR DEM and the least squares solution of the problem (1), in which it is possible to notice a rising trend with respect to the slant range coordinate. Such an artifact can be avoided by assuming the ALGAE approach, namely by reintroducing the null space with the proper weighting factor. Fig. (1), top right, shows the allowed null space for data-set under analysis. The shape of this component is clearly correlated with the error found in the least squares solution. As a consequence, the usage of the null space profile, conveniently weighted, provides a more accurate result than a simple de-trending of the solution, since it properly takes into account the actual ambiguity of the problem. The advantage over the rectilinear de-trending can be easily understood from Fig. (1), bottom left, where the same null space is displayed after removing its rectilinear component, in such a way as to emphasize local inhomogeneities. Final ALGAE result is shown in Fig. (1), bottom right. Residual errors are visible, whose dynamic may be assessed in about $\pm 15m$. Such errors are characterized by a large spatial decorrelation length and are correlated with the flight direction. These features clearly indicate that terrain elevation errors are the result of uncompensated non constant phase terms, due to the residual platform motion.

2. REFERENCES

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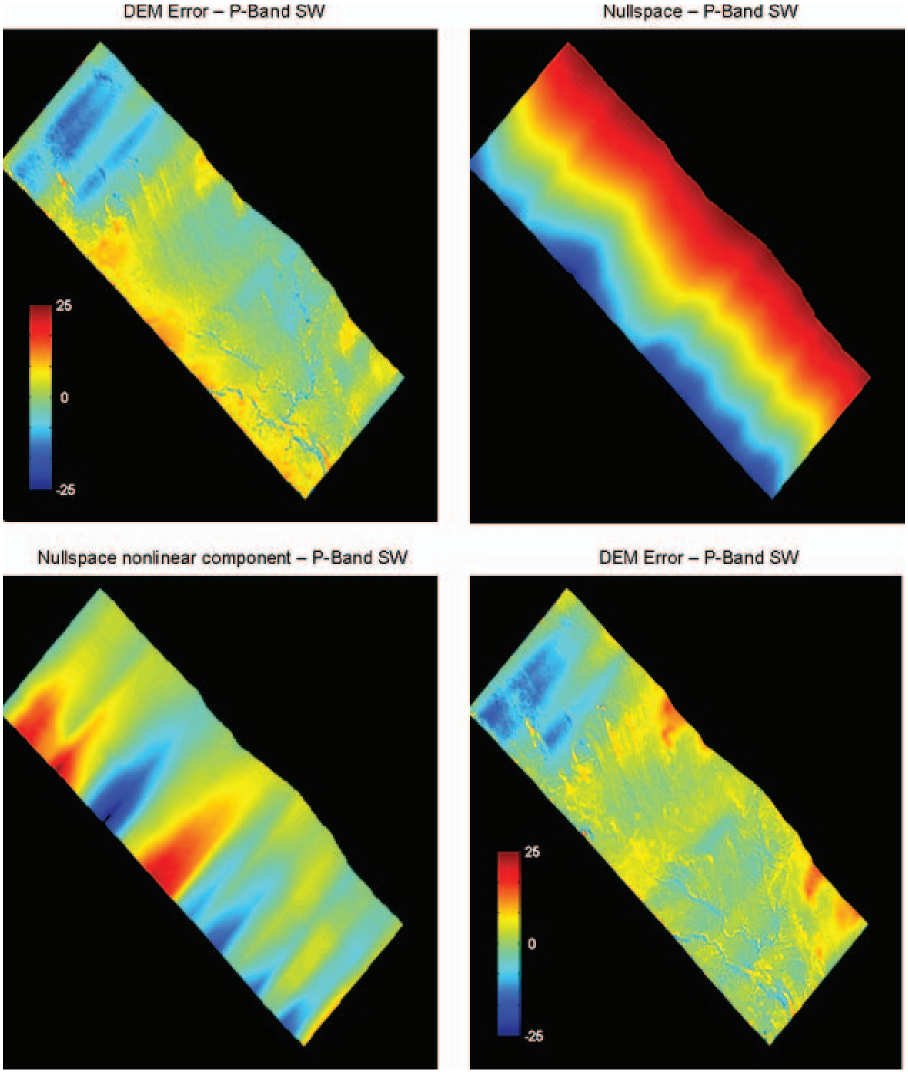

Fig. 1. Top left: elevation error $[m]$ between the LIDAR DEM and the Least Squares Solution. Top right: Allowed null space component. Bottom left: Allowed null space non-rectilinear component. Bottom right: elevation error $[m]$ between the LIDAR DEM and the ALGAE Solution.