FROM LIDAR SIGNALS TO AEROSOL MICROPHYSICS BY REGULARIZATION

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1. INTRODUCTION

Aerosols in the Earth's atmosphere are having a wide range of influences on local and global climate and weather, such as on clouds and precipitation. They are important to understanding the chemical processes happening in the troposphere and the stratosphere, see [1]. Also, the direct and indirect effects of aerosols are the two largest contributions to the total uncertainty of the radiative forcing, which roughly describes the difference between incoming and outgoing energy of the tropopause, and thus can be used in researching global climate change. Several methods exist to measure the concentration and attributes of aerosol particles, like the refractive index to estimate the single scattering albedo, or for instance aerosol size. One such method is the lidar (light detection and ranging).

2. AEROSOL OPTICAL PARAMETERS BY REGULARIZATION

We consider here the stable algorithm for retrieving the aerosol extinction coefficient from measured lidar signals. Suggested by Ansmann *et al.* [2] the aerosol extinction coefficient profile $\alpha^{\mathrm{aer}}(\lambda_{\mathrm{L}}, \cdot)$ at the emitted laser wavelength λ_{L} by using the Raman lidar technique can be formulated as

$$\alpha^{\mathrm{aer}}(\lambda_{\mathrm{L}}, z) = \frac{\frac{\mathrm{d}}{\mathrm{d}z}\ln\left[\frac{N(z)}{z^2 P(\lambda_{\mathrm{R}}, z)}\right] - \alpha^{\mathrm{mol}}(\lambda_{\mathrm{L}}, z) - \alpha^{\mathrm{mol}}(\lambda_{\mathrm{R}}, z)}{1 + \left(\frac{\lambda_{\mathrm{L}}}{\lambda_{\mathrm{R}}}\right)^{\kappa}} \tag{1}$$

where $P(\lambda_{\mathrm{R}}, z)$ is the power received from distance (range) z at the Raman wavelength λ_{R}, $N(z)$ is the atmospheric number density of the Raman scatterer, κ is the Ångström exponent and α^{mol} is the molecular extinction coefficient at λ_{L} and λ_{R}. Here we assume a wavelength dependence of the aerosol extinction $\alpha^{\mathrm{aer}}(\lambda, z) \propto \lambda^{-\kappa}$ and consider those heights such that the lidar system overlap function is equal to unity.

2.1. Proposed Iterative Regularization

The aerosol extinction coefficient defined in (1) requires the calculation of the derivative of the logarithm of the ratio between the atmospheric number density and the range-corrected lidar-received power. The calculation of the derivative from measurement data is ill-posed in the sense that small fluctuations of the Raman signal can produce large fluctuations in the derivative and thus in the aerosol extinction profile. The derivative problem in retrieving of optical extinction coefficient profiles from lidar signals was treated in [3] where the calculation of the derivative can be seen as a direct problem. Here we calculate the derivative by the inversion technique, i.e. transforming the derivative term into the Volterra integral equation as follows

$$\int_{z_{\min}}^{z} x(s)\mathrm{d}s = \ln\left[\frac{z_{\min}^2 P(\lambda_{\mathrm{R}}, z_{\min}) N(z)}{z^2 P(\lambda_{\mathrm{R}}, z) N(z_{\min})}\right], \quad z \in [z_{\min}, z_{\max}] \tag{2}$$

where $z_{\min}$ and $z_{\max}$ are the minimum and maximum ranges for determining the derivative. The continuous problem (2) is discretized and reduced to a finite system of linear equations by the collocation method which is a kind of projection method.

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Indeed, pure projection methods into finite dimensional subspace X_n of dimension n act as regularization themselves without any additional filter where the dimension n serves as regularization parameter. Let $z_{\text{min}} = z_1 < z_2 < \cdots < z_n$ with $z_n < z_{\text{max}}$ be given collocation points with equally width. By the collocation method with an orthogonal basis $\{\varphi_j\}_{j=1}^n$ of X_n defined by a piecewise continuous function, i.e. $\varphi_j(s) = 1$ for $s \in [z_j, z_{j+1}]$ and $\varphi_j(s) = 0$ otherwise, the problem (2) is reduced to the following linear equation

$$L_{(n)} f = g \tag{3}$$

where $L_{(n)}$ is a $(n-1) \times n$ constant matrix depending on the discretization method and the width of the range and g is a $(n-1) \times 1$ vector depending on input data on the right hand side of (2). A stable solution of (3) can be obtained by the linear Levenberg-Marquardt method, i.e.

$$f_{k+1} = f_k + \gamma \left(I + \gamma L_{(n)}^T L_{(n)} \right)^{-1} L_{(n)}^T \left(g - L_{(n)} f_k \right), \quad k = 0, 1, \ldots \tag{4}$$

if it is stopped at a suitable termination index k_*. Here γ is a relaxation parameter and f_0 is an initial guess. In the presented work the termination index is chosen by the L-curve method, i.e. choose the parameter which corresponds to the regularized solution nearest to the corner point of the letter L. To select the suitable size of the subinterval corresponding to the dimension of the subspace, the L-curve method is applied again, where the curvature function is needed for locating the corner point, which depends on the parameter n, see [4] for details.

2.2. Application

Our algorithm is applied to the measured lidar signals obtained from CNR-IMAA lidar station in Potenza (760 m a.s.l., 40.6N, 15.72E) on August 30, 2007 at 18:06-18:18 UTC. The aerosol extinction coefficient profiles obtained by the developed algorithm are shown in Fig. 1 (right side) where we can see that the results are comparable with those computed by the algorithm from Potenza. Moreover, the standard error obtained by using the concept of Monte Carlo procedure is admissible. Fig. 1 (left side) shows the L-curves corresponding to their extinction coefficients where the selected parameters n_* are indicated by thick L-curve with associated point on the corner indicated by a magenta rhombus. In Fig. 1 (left hand side) on each subinterval with size n the plots of the norm of the regularized solutions versus the norm of the corresponding residual for all valid regularization parameters k are shown and the point on the corner is indicated by a black circle which is obtained by the curvature function.

3. AEROSOL MICROPHYSICAL PARAMETERS BY REGULARIZATION

We will discuss now an algorithm to extract information about the distribution of the particles from multiwavelength lidar data, or more specifically, from extinction and backscatter coefficients gained from the respective profiles.
For multiple wavelength lidar we can obtain the following equation, in this case for number distribution of the particles:

$$\Gamma(\lambda) = \int_{r_{\text{min}}}^{r_{\text{max}}} \pi r^2 Q_{\pi/\text{ext}}(r, \lambda, m) n(r) \mathrm{d}r \tag{5}$$

with the kernel functions Q_π for backscatter and Q_{ext} for extinction, respectively; m is the complex refractive index, λ the wavelength, $n(r)$ the size distribution, and r_{min} and r_{max} are sensible lower and upper bounds for the size of the particles.
In common measurement setups, we expect a lidar setup consisting of 3 backscatter wavelengths (at 355, 532 and 1064 nm) and 2 extinction wavelengths (at 355 and 532 nm). Thus, only five data points are available for the inversion process and they are collected in the vector $\Gamma(\lambda)$.
Equation (5) is an integral equation of first Fredholm kind, thus, an ill-posed problem requiring regularization. Much research has been done already in the area of solving this inverse equation. One approach that has been followed in [5, 6] makes use of spline collocation and truncated singular decomposition or spline collocation and pure iterative regularization. Also, to solve the nonlinear problem with unknown refractive index, the retrieval of the refractive index is handled by calculating solutions on a predefined grid and manually picking out solutions from that grid by criteria like the residual error. This is also done in the algorithm from [7], except that a Tikhonov method is used for the regularization here. The authors in [7] also employ a two-dimensional regularization approach which reduces extensive data postprocessing procedures.

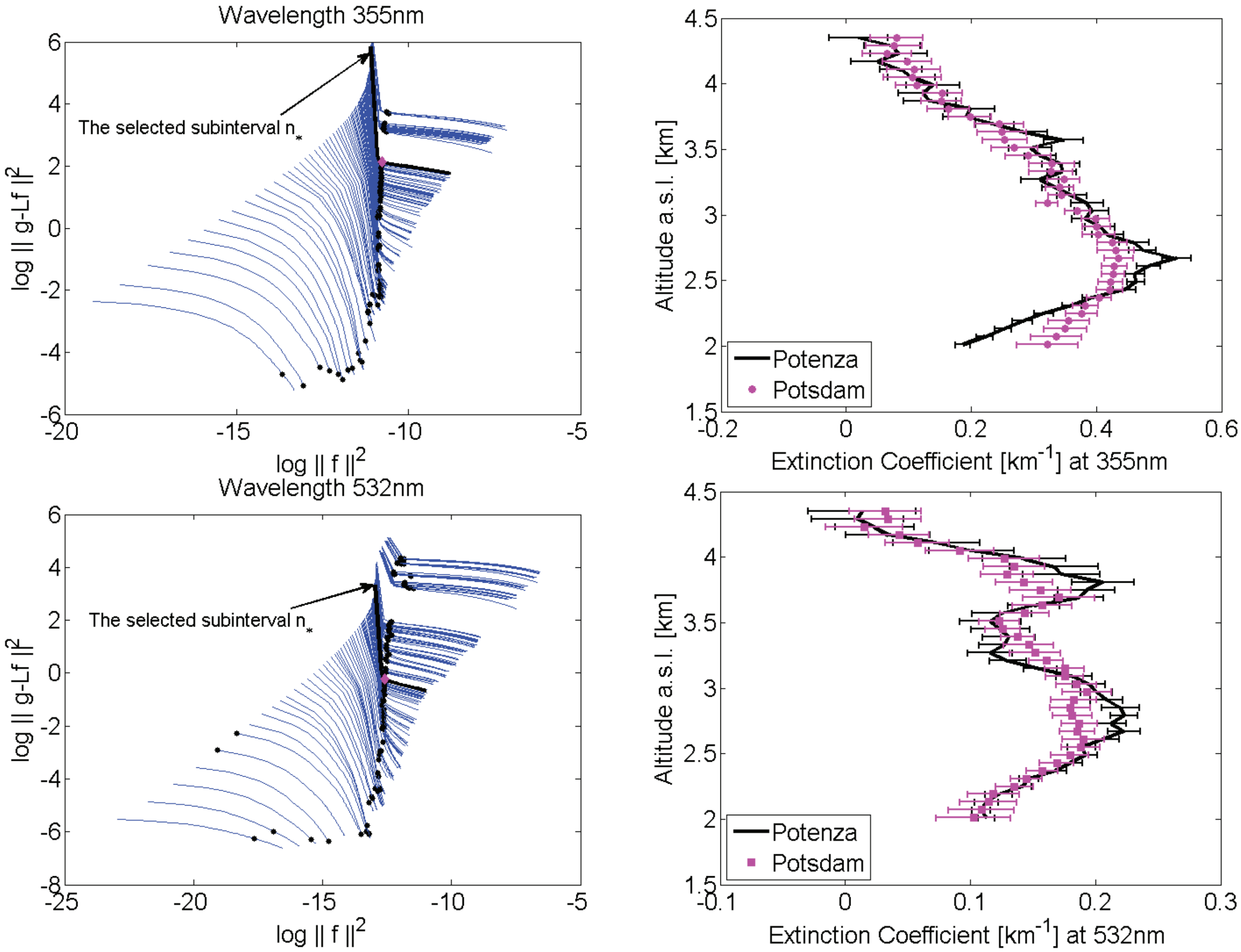

Fig. 1. Retrieval of Extinction coefficient profiles by two different methods (right side) and used L-curve parameter choice rule of the regularization method from Potsdam (left side).

3.1. Projected Iterative Regularization

Here, we choose a Padé regularization approach, which is defined by the operator, see [8] for details,

$$P^{(2,1)}x_i := x_{i+1} = x_i + \tau \sum_{j=0}^{\infty} \frac{1 + \tau \frac{\sigma_j^2}{3}}{1 - 2\tau \frac{\sigma_j^2}{3} + \tau^2 \frac{\sigma_j^4}{6}} \langle A^* y - A^* A x_i, u_j \rangle u_j.$$

This has the great advantage that the relaxation parameter τ can be arbitrarily chosen, see [6]. There is a strong connection between the choice of base points and the quality of the reconstruction of the volume distribution. To take that into consideration, the authors propose an algorithm that adapts the base points according to certain parameters. Indeed, it is easy to see how the solution is strongly smoothed out in areas with only a few base points, and sharp features require lots of base points in their vicinity to be reconstructed accurately. To respect that fact, our idea is to slide the base points according to some scheme. Thus, we propose to make the base points of the splines variable, sliding them towards the points that have more weight in the function. This can be done a certain number of times, always projecting the iteration in between to safeguard the non-negativity restriction of the distribution function. Of course, the iteration operator $P^{(2,1)}$ has to be recalculated after every base point change. After we get a final set of base points, we restart the Padé iteration with our last projected iterative result as starting value, and stop with the discrepancy principle, thus rendering the algorithm a regularization in the mathematical sense. Indeed, in our experiments, the discrepancy principle always stopped after the first iteration, hinting at the fact that the starting value is already close to the solution.

3.2. Application

The algorithm has also been applied to some measurement data gained on June 29, 2007 around 21:00 UTC, in Athens, Greece, during a forest fire event. The retrieval of the refractive index via our algorithm has been successfully validated in this measurement case via the chemical ISORROPIA II model (see [9]). While our algorithm retrieved a refractive index of $m = 1.386 + 0.006\text{i}$, see Fig. 2, the two retrieved modes had a refractive index of $m = 1.4059 + 0.006\text{i}$ for the coarse mode and of $m = 1.4062 + 0.006\text{i}$ for the fine mode calculated by the ISORROPIA II model.

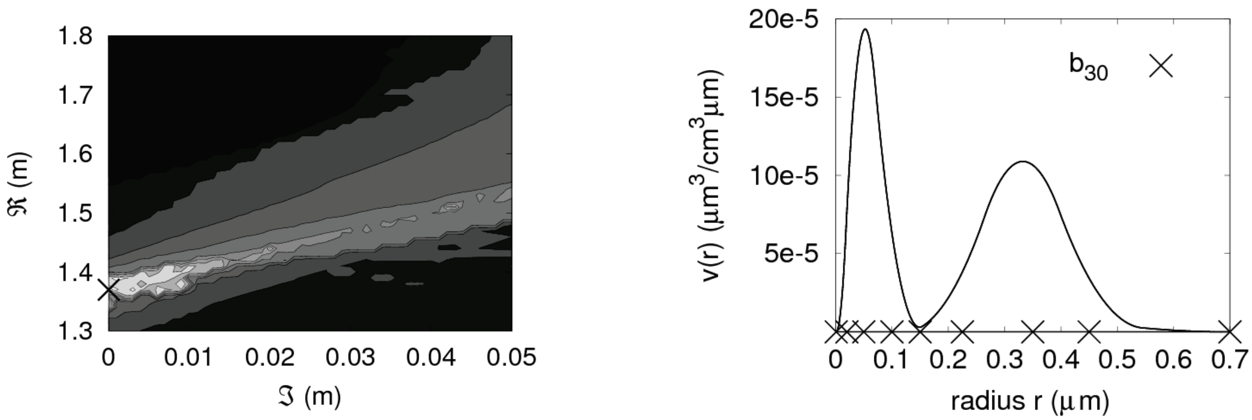

Fig. 2. Retrieval of the refractive index by the described grid technique (× area of the best indices) (left side) and retrieval of the bimodal volume distribution (right side) via the new projected method with variable base points for the measurement case of Athens (b_{30}: base points and solution (single line) after 30 iterations, × non-equidistant adaptive base point grid).

4. REFERENCES

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