HYPERSPECTRAL MATCHED FILTERS WITH FALSE ALARM MITIGATION

R. DiPietro and D. Manolakis [*]
MIT Lincoln Laboratory
244 Wood Street
Lexington, MA 02420-9185
`dmanolakis@ll.mit.edu`

T. Cooley and R. Lockwood
Air Force Research Laboratory
Space Vehicles Directorate
3550 Aberdeen Ave SE
Kirtland AFB, NM 87117

J. Jacobson
National Air and Space
Intelligence Center
Wright-Patterson AFB, OH

ABSTRACT

The adaptive matched filter is one of the most widely used algorithms for target detection in hyperspectral imaging applications [1]. However, due to the linearity of the decision surface, there are certain types of spectra that may lead to false alarms. In this paper, we exploit the geometrical structure of the matched filter and the replacement target model to develop a procedure which, when used in conjunction with the matched filter, provides a simple and effective tool for false alarm mitigation (FAM). The performance of MF with FAM detector is compared with that of the ENVI provided Mixture Tuned Matched Filter (MTMF$^{\text{TM}}$) algorithm.

1. INTRODUCTION

The matched filter (MF) detection algorithm is given by

$$y = D(\boldsymbol{x}) = \kappa(\boldsymbol{s} - \boldsymbol{\mu}_b)^T \boldsymbol{\Sigma}_b^{-1}(\boldsymbol{x} - \boldsymbol{\mu}_b) \tag{1}$$

where $\boldsymbol{s}$ is the spectral signature of the target, $\boldsymbol{\mu}_b$ and $\boldsymbol{\Sigma}_b$ are the mean and covariance of the background clutter, and $\boldsymbol{x}$ the pixel under test [2, 1]. The normalization constant κ, which is chosen so that $y = D(\boldsymbol{s}) = 1$, is given by

$$\kappa = 1/(\boldsymbol{s} - \boldsymbol{\mu}_b)^T \boldsymbol{\Sigma}_b^{-1}(\boldsymbol{s} - \boldsymbol{\mu}_b) \tag{2}$$

The MF is the optimum detector for the hypotheses: $H_0 : \boldsymbol{x} \sim N(\boldsymbol{\mu}_b, \boldsymbol{\Sigma}_b)$ and $H_1 : \boldsymbol{x} \sim N(a\boldsymbol{s}, \boldsymbol{\Sigma}_b)$ for $a > 0$ (unknown). In practice, the mean and covariance of the background are estimated from the data (adaptive MF).

To evaluate the performance of (1) we consider the standard replacement target model defined by

$$\boldsymbol{x} = a\boldsymbol{s} + (1 - a)\boldsymbol{v}, \quad 0 \le a \le 1 \tag{3}$$

where a is the target fill factor. If the target variability is specified by $\boldsymbol{s} \sim N(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$, the mixed spectrum (3) follows a normal distribution with mean and covariance given by

$$\boldsymbol{\mu}(a) = a\boldsymbol{\mu}_t + (1 - a)\boldsymbol{\mu}_b \tag{4}$$

$$\boldsymbol{\Sigma}(a) = a^2\boldsymbol{\Sigma}_t + (1 - a)^2\boldsymbol{\Sigma}_b \tag{5}$$

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To facilitate our analysis we consider the following linear transformation

$$\tilde{\boldsymbol{x}} = T(\boldsymbol{x}) = \frac{1}{\Delta_b}\boldsymbol{\Sigma}_b^{-1/2}(\boldsymbol{x} - \boldsymbol{\mu}_b) \tag{6}$$

where Δ_b is the Mahalanobis distance

$$\Delta_b^2 = (\boldsymbol{\mu}_t - \boldsymbol{\mu}_b)^T \boldsymbol{\Sigma}_b^{-1}(\boldsymbol{\mu}_t - \boldsymbol{\mu}_b) \tag{7}$$

This transformation "spherizes" the background distribution (whitening) and normalizes the distance between target and background distribution centers to one. This process is illustrated in Figure 1. The MF in the whitened space is which is the projection of the pixel under test to the vector connecting the centers of target and background distributions.

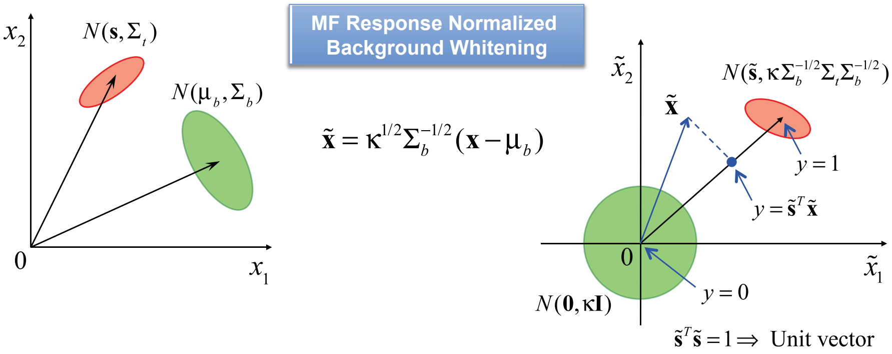

Fig. 1. Spherical background transformation.

$$\tilde{y} = \tilde{\boldsymbol{s}}^T \tilde{\boldsymbol{x}} \tag{8}$$

2. FALSE ALARM MITIGATION

The basic idea of proposed false alarm mitigation approach is illustrated in Figure 2. We note that all spectra with tips on the hyperplane perpendicular to $\tilde{s}$ at the point determined by the response of the matched filter yield the same response. However, only the ones "close" to $\tilde{s}$ are likely to come from a target spectrum.

The likelihood of a spectrum $\boldsymbol{x}$ to originate from a mixture of the target spectrum with background is specified by a normal distribution with mean and covariance given by (4) and (5). The fill factor a is estimated by the output of the normalized matched filter and the target mean by $\boldsymbol{s}$. The covariance of the mixed pixel distribution is approximated by the background distribution. This is some kind of arbitrary decision, but is seems to work reasonably well in practice. For a normal distribution, the likelihood is inversely proportional to the Mahalanobis distance (physical space)

$$y_{MD} = (\boldsymbol{x} - \boldsymbol{\mu}(\hat{a}))^T \boldsymbol{\Sigma}_b^{-1}(\boldsymbol{x} - \boldsymbol{\mu}(\hat{a})) \tag{9}$$

Therefore, we can use (9) to decide whether a matched filter hit is a target or a false alarm. The result is a two-threshold decision making process, which is illustrated in Figure 3. The target pixels are shown in red color, whereas the background pixels are shown in black color. Figure 4 shows results for the same data using the ENVI mixture tune matched filter (MTMF$^{\text{TM}}$) [3]. The MTMF requires the selection of a "spectrally homogeneous" area to estimate a noise covariance matrix, which is used as a surrogate for the target covariance matrix. This is a difficult task, which essentially makes the algorithm "user-dependent". More details and additional shortcomings of the MTMF will be presented at the full paper.

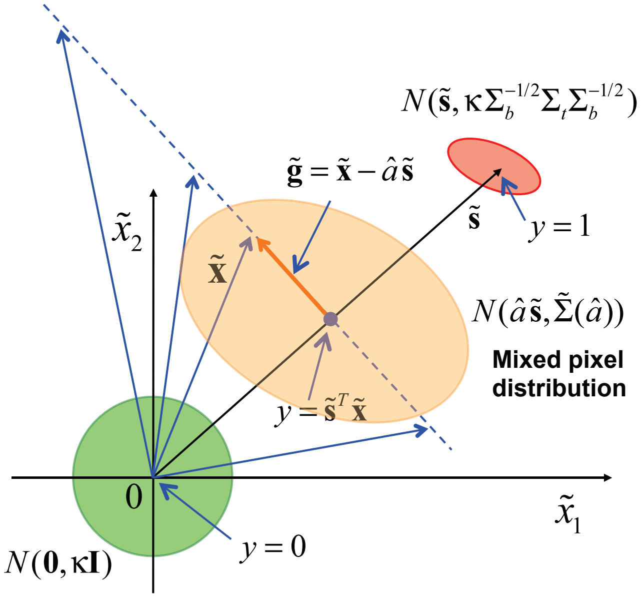

Fig. 2. Geometrical explanation of false alarm mitigation.

3. SUMMARY

We presented a MF detector with false alarm mitigation capability. Based on our investigations the proposed MF-FAM algorithm provides the same or better performance than the more complicated ENVI MTMF.

4. REFERENCES

[1] D. Manolakis, R. Lockwood, T. Cooley, and J. Jacobson, “Is there a best hyperspectral detection algorithm?,” 2009, vol. 7334, p. 733402, SPIE.

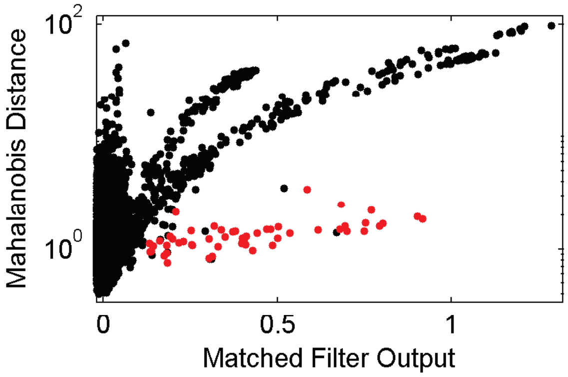

Fig. 3. Results of MF with false alarm mitigation.

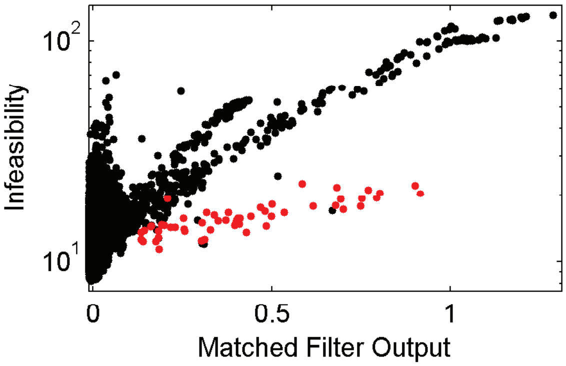

Fig. 4. Results of ENVI mixture tune matched filter.

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[3] J. W. Boardman, "Leveraging the high dimensionality of aviris data for improved sub-pixel target un- mixing and rejection of false positives: mixture tuned matched filtering," in *Summaries of the Seventh Annual JPL Airborne Geoscience Workshop*, R. O. Green, Ed., Pasadena, California, 1998, JPL Publication 97-21, vol. 1, p. 55.