A NOVEL APPROACH FOR REDUNDANT INTEGRATION OF FINITE DIFFERENCES AND PHASE UNWRAPPING ON A SPARSE MULTIDIMENSIONAL DOMAIN

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1. INTRODUCTION

In SAR interferometry, and differential and persistent scatterer interferometry (but also in other technical fields), phase unwrapping and reconstruction of elevations and displacements are key problems [1-4]. In both cases the starting data are differential measurements (finite differences between neighboring points), typically obtained, in the first case, by assuming that the difference of phases in neighboring points are smaller than half a cycle, in the second case, by maximizing the so-called temporal coherence. Therefore, the two problems can be formulated identically as the problem of reconstructing a function given preliminary estimates of its finite differences, which in general are not consistent, i.e. their integration depends on the integration path. The only difference in these two problems is in the allowed corrections to the preliminary estimates: in case of phase unwrapping the corrections should be discrete (an integer multiple of 2π), while in the other cases the corrections can take any values. This makes the phase unwrapping problem more difficult, with a lot of literature produced on this topic [5-11].

In SAR interferometry (including differential and persistent scatterer interferometry) often a stack of images acquired at different times, baselines or frequencies are available. Usually, those data are processed independently or at most working on one domain at once [12]. Some techniques for 3D phase unwrapping have been proposed, [13] and [14], but they need a preliminary data calibration to compensate atmosphere and orbital phase contributions.

Starting from the consideration above, we propose a general formulation for the integration of finite differences and phase unwrapping problems. We propose an approach (comprising standard phase unwrapping techniques as special cases), that allows obtaining a robust and accurate solution by exploiting redundant information, obtained working with differences between not only nearest neighoring pixels. Moreover, the proposed formulation allows to exploit multi-dimensional information (2D space + time, multi-frequency, multi-baseline, etc.), and to integrate external information if available (e.g. GPS). In all cases the solution can be efficiently obtained by solving a linear or a quadratic programming (LP or QP) problem.

Based on the proposed general formulation, we propose and validate different solutions for:

- redundant 2D and 3D phase unwrapping and finite difference integration
- joint phase unwrapping of a temporal series of interferograms (e.g. in persistent scatterer interferometry) robustly with respect to atmospheric or orbital contributions in the phase
- joint phase unwrapping of multi-baseline and multi-frequency (band-splitted wide-band) SAR data.

The validation tests obtained on real SAR data confirm the validity of the approach. Computational efficient algorithms exist and the redundant arc differences integration integration/phase unwrapping method was integrated in our production chains and extensively validated and used also for massive productions.

2. METHOD

Consider the problem of reconstructing a function on a set of sparse points in a multidimensional domain from preliminary estimates for the values of linear combinations (in the simplest case neighboring point differences) of the function at the points to be reconstructed. This reconstruction problem is formulated as the inversion of a linear system of equations, where the constraint matrix defines the linear relations between the function values at different points, and the right hand side terms represent the corresponding estimates (and in case the constant term of the linear combination). In the cases of interest the system is over-determined and the preliminary estimates are inconsistent. The solution can be determined by minimizing, according to a given metric (e.g. L1, L2), the residuals of the different determinations. The problem can be efficiently solved with quadratic and linear programming (QP and LP) solving algorithms in the case of the norm L1 and L2, respectively. The classical problems of SAR interferometry, i.e. phase unwrapping and mean velocity/elevation reconstruction are included in this formulation.

The L2 metric is more appropriate when a normal distribution of the errors it is expected, whereas the L1 norm guarantees solutions more robust to outliers, avoiding error spreading, and is therefore very useful for the applications examined in this paper. In particular, when each of the considered preliminary estimates refers to a difference between two of the (sparse) points in the multidimensional domain, it can be demonstrated that the constraint matrix is totally unimodular. This property is important because an LP problem with totally unimodular constraints matrix and integer parameters has integer solutions, as expected in phase unwrapping to obtain integer multiples of 2π. And remains valid independently on how many pairs of points we consider for the differences. When considering finite difference integration problems, as for example in mean velocity and elevation reconstruction in persistent scatterer interferometry, the problem has not anymore integer parameters and the solution is not integer, which is expected and correct.

In our approach, using a very redundant set of preliminary estimates of differences between nearby points, and not only the nearest neighboring points, is very simple and correspond to add equations in the considered system, making it more overdetermined. As a result, phase unwrapping and mean velocity/elevation reconstruction are more accurate and robust to noise and outliers.

If a temporal series of interferometric phases are available, in the proposed formulation a 3D (2D space + 1D time) redundant problem can be straightforwardly written. We already proposed some years ago [13], an analogous extension to a 3D structure, although less generic and not allowing for a high redundancy. However, the straightforward extension to 3D, with differences between phases acquired at different times, require calibrating the interferometric phases in order to reduce atmospheric and orbital systematic errors.

The proposed general formulation allows for more flexible of the available multidimensional information. We propose a solution for multitemporal phase unwrapping of a series of interferograms (e.g. in persistent scatterer interferometry) that overcomes the phase calibration problem by considering double-differences of phases such that atmospheric and orbital contributions cancel.

Finally, we propose also solutions for phase unwrapping of multi-baseline and multi-frequency (band-splitted wide-band) SAR data, based on different scaling factors for the information layers corresponding to different baselines or frequencies.

3. EXAMPLES AND VALIDATION

The proposed method was validated and integrated in our existing production chain, with successful use also in massive productions. The obtained results will be described. In this preliminary abstract we report one test relative to the most difficult finite difference integration problem, the phase unwrapping. We considered two stacks of ENVISAT data over Abruzzo e Campania regions (Italy). We compared different phase unwrapping strategies on these data: based on nearest neighbors defined by the Delaunay triangulation [11], based on redundant differences between nearby points (not only nearest neighbors), and jointly unwrapping multi-temporal phases. In order to determine the quality of the results, we applied a consistency check. This is based on the fact that the results should not depend on the integration path in the temporal domain, i.e. for each triplet of images the temporal circuitation should be null. The consistency check is verified by definition for multi-temporal phase unwrapping. In fact, the consistency is one of the conditions enforced in this case, which help finding a better solution.

In Figure 1 and Figure 2 the inconsistencies of Delaunay triangulation and redundant arcs phase unwrapping of three ENVISAT acquisitions over Abruzzo region are shown, respectively. It can be noted that the usage of redundant information allows drastically reducing phase unwrapping errors present with respect to the standard case. Other validation tests will be presented in the final paper.

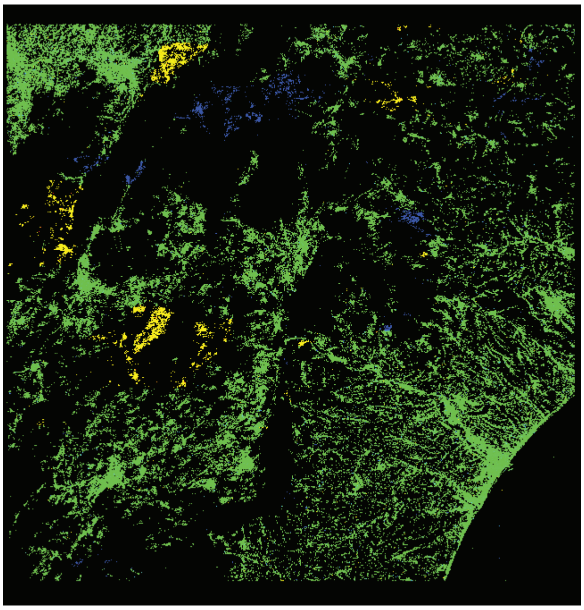

Figure 1: Inconsistencies of Delaunay triangulation phase unwrapping over Abruzzo region (Italy) relative to the ENVISAT acquisitions of: (a) 2004/05/26, (b) 2004/06/30, (c) 2004/08/04. More than 7 % of points show inconsistencies.

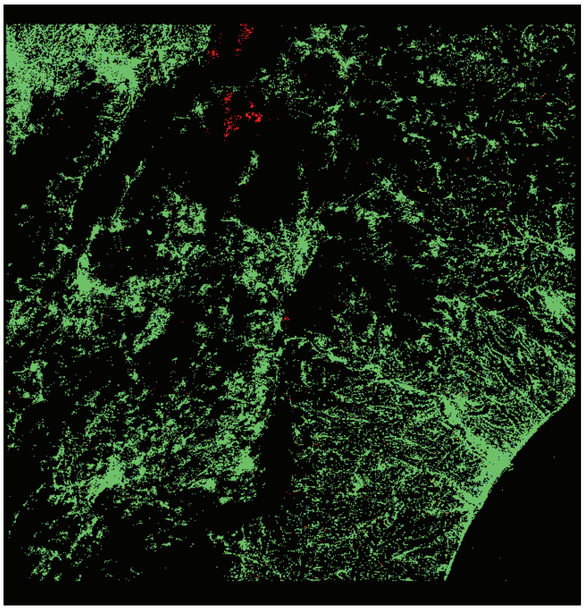

Figure 2: Inconsistencies of redundant 2D phase unwrapping over Abruzzo region (Italy) relative to the same data of Fig. 1. About 2 % of points show inconsistencies

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