A comparison method for THz measurements using VNA and TDS

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***Abstract*—A method is described for direct comparison of dielectric measurements obtained by a vector network analyzer and a time domain spectrometer. The method employs a material that can be inserted into a waveguide for VNA measurements or contained in a cell for TDS measurements.**

I. INTRODUCTION

VECTOR network analyzers (VNA) and time-domain spectrometers (TDS) occupy different and separate spheres of THz measurements, in that VNAs are configured for device characterization using hollow rectangular metallic waveguides, whereas spectrometers measure primarily material properties and operate in free-space. The bandwidth of VNA instruments has recently expanded to above 1 THz, and is set to extend further, resulting in a significant overlap between VNA and TDS spectral ranges. Nevertheless, VNA and TDS measurements remain distinct, divided by both areas of application and measurement methodologies.

Some overlaps exist, in that VNAs have been occasionally used in free-space quasi-optical setups as a platform for THz spectroscopy [1,2]; and the dielectric constants of materials measured by such systems were shown to be similar to those obtained by TDS. In this paper we describe a method which makes possible a rigorous comparison of permittivity measurements obtained by VNA and TDS, whereby both devices are used in their customary configurations.

II. EXPERIMENTAL

Since VNA is designed for operation via rectangular hollow waveguides, whereas TDS operates in free space, it was decided to accomplish the intercomparison by using a material capable of being measured by both modalities.

For free-space TDS measurements the material was contained in a spectrophotometric quartz cell (VWR® Spectrosil) whose thickness was 5.00 mm. The reference signal was recorded with an empty cell. The TDS was a lab-constructed instrument, and had a standard configuration using a biased photoconductive emitter with electro-optic detection and 4 parabolic mirrors.

In order to measure the S-parameters of the same material using VNA, it was inserted inside sections of high-precision WR-05 waveguide. Three different lengths of waveguide section were used, with nominal lengths of 2.5 mm, 5.0 mm and 7.5 mm. The measurements were performed using a Keysight Technologies PNA-X VNA connected to a pair of Virginia Diodes Inc waveguide extender heads fitted with standard WR-05 waveguide test ports designed for the frequency band 140-220 GHz.

Two types of material were studied: pure petroleum jelly (PJ); and petroleum jelly mixed with carbon powder (PJC). Petroleum jelly is a mixture of alkanes, and is therefore non-polar with high THz transparency and low refractive index. It was was chosen for its thermo-mechanical properties, in particular for its low melting point and semi-solid character at room temperature. Carbon powder with grain size <1 μm was mixed with petroleum jelly in the ratio of 10% by volume. Carbon is a conductor; therefore the resulting mixture had a higher refractive index and stronger THz absorption than PJ.

In the TDS the frequency measurement band (due to dynamic range limitations) was 0.1-2.6 THz for PJ and 0.1-0.9 THz for PCJ; whereas the frequency band of the VNA was 140-220 GHz. Therefore the frequency overlap between the two instruments was over the whole of the VNA range, but at the lower limit of operational range of the TDS.

III. RESULTS

To obtain the optical parameters from TDS measurements, the time-domain data was converted to frequency domain by applying Fast Fourier Transform (OriginPro™ 2015). The frequency-dependent (ω) refractive indices (n) and absorption coefficients (α), and the real and imaginary permittivities (ε' & ε'') were then calculated using the standard procedure [3]:

$$n(\omega) = 1 + \frac{(\phi_{cell} - \phi_{sample})\, c}{\omega\, d} \quad (1)$$

$$T(\omega) = 1 - \frac{(n-n_{cell})^2 + k^2}{(n+n_{cell})^2 + k^2} \quad (2)$$

$$k(\omega) = \frac{\alpha c}{2\omega} \quad (3)$$

$$\alpha(\omega) = -\frac{2}{d} \ln\left(T\, \frac{E_{sample}}{E_{cell}}\right) \quad (4)$$

$$\varepsilon = \varepsilon' + i\varepsilon'' = (n + ik)^2 = (n^2 + k^2) + i(2nk) \quad (5)$$

where E and ϕ are respectively amplitude and phase of the THz signal; d is the optical path through the cell, d = 5.00 mm; and n_{cell} is the THz refractive index of cell material, i.e. quartz, $n_{cell} \cong 1.95$.

The refractive indices and absorption coefficients of petroleum jelly and jelly-carbon mixture obtained by TDS are shown respectively in Figures 4 and 5. It is seen that, as expected, absorption is much higher in the carbon-containing material, limiting the spectral range of the measurement; the refractive index is also significantly higher. The values of refractive index, absorption coefficient, and the real and imaginary permittivities for both pure petroleum jelly and carbon-jelly mix are listed in Table 1 together with their expanded uncertainties. The TDS measurement uncertainty may be assumed to be equal to double the value of the standard deviation (95% confidence).

As seen in Figures 4 and 5, the frequency measurement band of the TDS, due to dynamic range limitations, was 0.1

THz to 2.6 THz for petroleum jelly and 0.1 THz to 0.9 THz for the jelly-carbon mixture. In contrast, the VNA in the study operated in the frequency band of 140 GHz to 220 GHz. Therefore the frequency overlap between the two instruments was over the whole of the VNA range, but at the lower limit of operational range of the TDS.

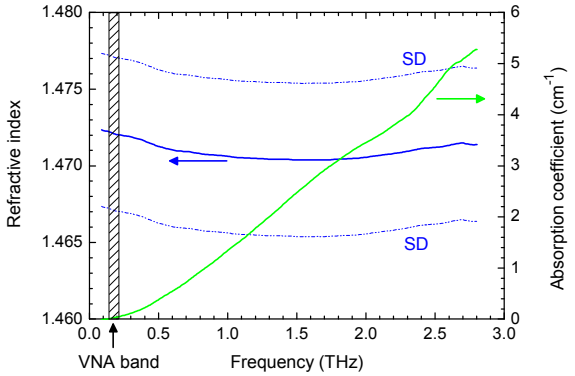

Fig. 4. The refractive index and absorption coefficient of pure petroleum jelly measured by TDS. Dashed lines indicate standard deviation of the refractive index. The standard deviation of absorption falls within the line thickness.

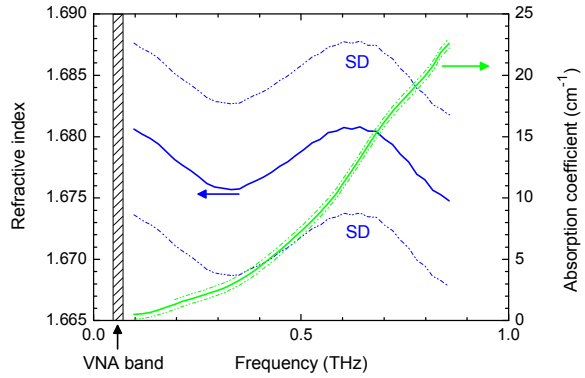

Fig. 5. The refractive index and absorption coefficient of petroleum jelly mixed with carbon powder (~10% carbon by volume) measured by TDS. Dashed lines indicate standard deviation.

The transmission coefficient measurements obtained using the VNA were used to compute the complex relative permittivity. The conversion method used for permittivity computation in this paper is the *Iterative One-Parameter Method*, described below [4].

The algorithm requires only transmission S-parameters and computes iteratively the relative complex permittivity for non-magnetic materials [22]. It is based on the following set of equations:

$$S_{21} = \frac{T(1-\Gamma^2)}{1-\Gamma^2 T^2}, \tag{6}$$

where

$$\Gamma = \frac{\gamma_0 - \gamma_1}{\gamma_0 + \gamma_1} \tag{7}$$

and

$$T = e^{-\gamma_1 L} \tag{8}$$

Also,

$$\gamma_0 = \sqrt{k_c^2 - k_0^2} \tag{9}$$

and

$$\gamma_1 = \sqrt{k_c^2 - \varepsilon_r k_0^2} \tag{10}$$

where k_c is the waveguide cut-off wave-number and k_o is the wave-number in air. The set of equations shown above is solved iteratively starting with the initial guess for permittivity (ε_r) as described in [4].

The results obtained by TDS and VNA measurements were compared. Since multiple measurements of each material were performed, mean values were used, and the uncertainties were defined as equal to two standard deviations (95% confidence). It was found that in all cases the data scatter was the dominant contribution to the estimated uncertainty. Table 1 summarises the results obtained by VNA and TDS, including the mean values and uncertainties.

TABLE 1.
Complex permittivities of petroleum jelly (PJ) and petroleum jelly with carbon powder (PJC) measured by VNA and TDS.

	PJ		PJC	
	VNA	TDS	VNA	TDS
ε’	2.132±0.031	2.167±0.028	2.551±0.156	2.815±0.044
ε”	0.006±0.008	0.002±0.002	0.031±0.006	0.050±0.030

Table 1 shows good agreement for all results with the exception of the real part of the complex permittivity of PJC. This may be attributed to the graininess of this material, which causes imperfect filling of the waveguide.

IV. Summary

A technique for the inter-comparison of dielectric measurements performed by VNA and TDS was demonstrated and validated. The technique has the advantage of employing both instruments in their standard configurations, i.e. VNA via a hollow metallic waveguide and TDS in free-space. The reliability of the technique was found to be critically dependent on the choice and preparation of the material under test. For greater accuracy, homogeneous, semi-solid materials should be used. The measured complex permittivity values obtained using the VNA and TDS showed good agreement.

Acknowledgement

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