# Terahertz Single Frequency Adaptive Holography with Large Depth of Focus

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***Abstract*—An adaptive focusing (AF) algorithm based on minimum-entropy (ME) method is proposed for THz single frequency holography to improve the depth of focus for practical image applications. Experiments in 0.2THz band were performed to confirm the effectiveness of the proposed algorithm.**

## I. INTRODUCTION

THz imaging has potential applications in various domains, such as nondestructive materials detection and security inspection. In single frequency THz holography, the echoed fields are collected by the pan scanning of the quasi-optics transceiver over a planer aperture, and then the 2D images are reconstructed with a fixed restoration range distance [1], which means the targets are assumed to be located in a plane and with a known and fixed distance. For practical applications, targets usually have surfaces with variant range distances, and the conventional single frequency holography no longer works due to its narrow depth of focus. In this paper, an adaptive focusing method based on the minimum entropy concept was proposed to realize THz single frequency holography with large depth of focus.

## II. RESULTS

Supposing an ideal point target is located at ($x_i$, $y_i$, $z_i$ ) with an “scattering coefficient” $\sigma_i$. The THz transceiver transmitting the Gaussian beam with its minimum beam waist radius $w_0$ at the scan plane $z$=0. The restoration range distance is selected to be $z_0$. Here, $z_0$ maybe not equal to $z_i$ which lead to the defocus of the image. The analytical expression of the defocused 2-D point spread function (PSF) is derived as:

$$\left|\tilde{s}_i(x,y)\right| = \sigma_i \frac{w_0^2}{w(z_i)\,w(z_i-z_o)} \cdot \exp\left\{-\frac{2\rho^2(x_i,y_i)}{[w(z_i-z_o)]^2}\right\} \tag{1}$$

Here $w(z_i-z_o)$ is the diffused waist radius of the transmitting beam at the plane $z_i-z_o$. This means the image defocus in a way like the diffusion of the Gaussian beam. When restoration range distance $z_0$ is exactly equal to target distance $z_i$, the best focused image can be obtained.

For targets having a surface with variant range distance $z(x,y)$, image reconstruction with fixed restoration distance $z_o$ is no longer effective. However, as long as the surface distribution can be approximated by an estimation function $z_e(x,y)$ accurately enough, BP algorithm can be applied to reconstruct the well-focused single frequency holography image.

$$\tilde{s}(x,y) = \iint s(x',y')\exp\left[j2kz_e(x,y)\right] \cdot \exp\left\{\frac{j4kz_e(x,y)}{4z_e^2(x,y)+\left(kw_0^2\right)^2}\left[(x-x')^2+(y-y')^2\right]\right\}dx'dy' \tag{2}$$

To get the optimum evaluation of $z_e(x,y)$, entropy is introduced to evaluate the focusing quality of THz images.

The entropy of the image $\left|\tilde{s}(x,y)\right|$ is defined as in [2]:

$$P = \int_x\int_y -I(x,y)\ln\left[I(x,y)\right] \tag{3}$$

where $I(x,y)$ is the power normalized image defined as :

$$I(x,y) = \frac{\left|\tilde{s}(x,y)\right|^2}{\int_x\int_y\left|\tilde{s}(x,y)\right|^2} \tag{4}$$

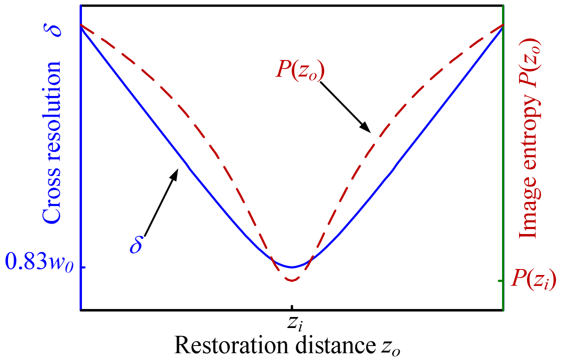


**Fig. 1** Cross resolution and Image entropy of THz image

Fig.1 shows the resolution and the entropy for the image of a point target located at ($x_i$, $y_i$, $z_i$) with different restoration distance $z_0$. It’s seen that when $z_0 = z_i$, the best resolution and the minimum entropy of the image achieved at the same time.

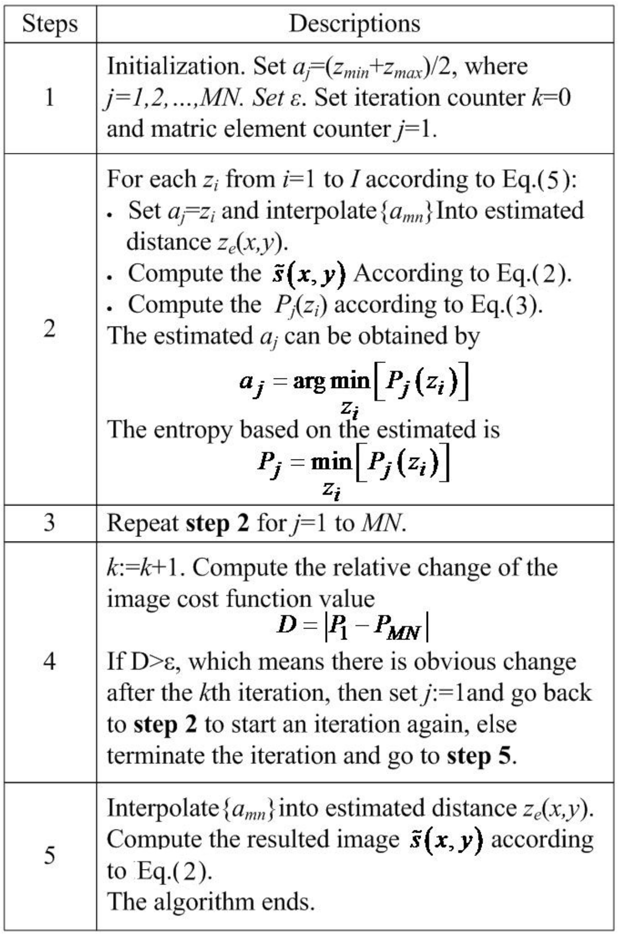
Therefore, entropy can be utilized to evaluate the focus quality for the image reconstruction in THz single frequency holography. Better focus results in a sharper image, and thus smaller entropy.

Based on the minimum entropy concept and the stage by stage approaching (SSA) method [3] for searching the optimal solution, a ME-AF algorithm was developed to realize the adaptive focusing in THz single frequency holography.

Supposing the target to be imaged has a surface with variant range distance $z(x,y)$ defined over the cross range x-y plane in the domain [-Δx/2, Δx/2] ×[-Δy/2, Δy/2]. To estimate $z(x,y)$ based on minimum entropy method, one can define a function $z_e(x,y)$ which is modeled as the interpolation of a matrix$\{a_{mn}\}_{M\times N}$, which is sampled with intervals Δx/M and Δy/N. Then, by discretizing the range distance, $a_{mn}$ can be chosen as :

$$z_i = z_{\min} + (i-1)\,dz,\ i = 1,2,\cdots,I \tag{5}$$

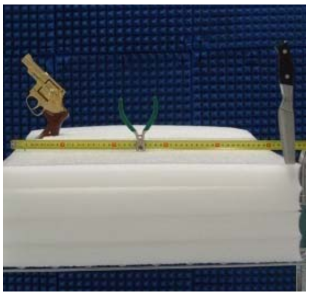
The main flow chart of the ME-AF is shown in Fig.2. The whole procedure of the reconstruction algorithm consists of five main steps.



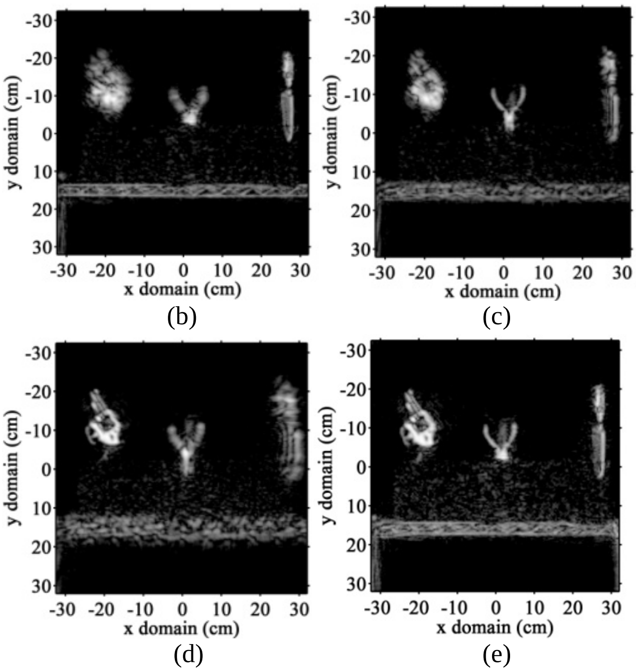
| Steps | Descriptions |
|---|---|
| 1 | Initialization. Set $a_j=(z_{min}+z_{max})/2$, where $j=1,2,\ldots,MN$. Set $\varepsilon$. Set iteration counter $k$=0 and matric element counter $j$=1. |
| 2 | For each $z_i$ from $i$=1 to $I$ according to Eq.(5):<br>- Set $a_j=z_i$ and interpolate $\{a_{mn}\}$ Into estimated distance $z_e(x,y)$.<br>- Compute the $\tilde{s}(x,y)$ According to Eq.(2).<br>- Compute the $P_j(z_i)$ according to Eq.(3).<br>The estimated $a_j$ can be obtained by<br>$a_j = \arg\min_{z_i}\left[P_j(z_i)\right]$<br>The entropy based on the estimated is<br>$P_j = \min_{z_i}\left[P_j(z_i)\right]$ |
| 3 | Repeat **step 2** for $j$=1 to $MN$. |
| 4 | $k$:=$k$+1. Compute the relative change of the image cost function value<br>$D = \lvert P_1 - P_{MN}\rvert$<br>If D>ε, which means there is obvious change after the $k$th iteration, then set $j$:=1and go back to **step 2** to start an iteration again, else terminate the iteration and go to **step 5**. |
| 5 | Interpolate $\{a_{mn}\}$ into estimated distance $z_e(x,y)$.<br>Compute the resulted image $\tilde{s}(x,y)$ according to Eq.(2).<br>The algorithm ends. |

**Fig. 2** The main flow chart of the ME-AF

To investigate the effectiveness of the algorithm, experiments on three metal objects were performed in 0.2THz. Three targets are located at 0.33m, 0.53m and 0.83m, respectively, with maximum range depth about 0.5m, as shown in Fig.3(a). Firstly, images were reconstructed based on conventional holography with different restoration distances $z_0$=0.33m, 0.53m and 0.83m. Fig.3(b), (c) and (d) show the images which are only focused individually on knife, pliers and pistol. Fig.3(e) shows the reconstructed image based on the proposed ME-AF algorithm. It's seen that all the three targets can be focused which verify the effectiveness of the proposed adaptive focusing method.



(a)



(b) (c)

(d) (e)

**Fig. 3** (a)optical image of the target. (b), (c) and (d) are THz images reconstructed based on conventional holography with restoration distance 0.33m, 0.53m and 0.83m, respectively. (e) THz image by ME-AF algorithm.

## III. SUMMARY

This Paper proposed a ME-AF algorithm for THz single frequency holography to achieve large depth of focus. The effective of the algorithm is demonstrated by experimental results in 0.2THz.

## REFERENCES

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