Study on Radiation Source with Negative-index Materials

D. Li[1], Y. Wang[2], Y. Wei[2], Z. Yang[2], S. Miyamoto[3]
[1]Institute for Laser Technology, 2-6 yamadaoka Suita, Osaka, 5650871 Japan
[2]University of Electronic Science and Technology of China, Chengdu 610054 China
[3]University of Hyogo, Ako, Hyogo 6781205 Japan

Abstract— **We present a novel radiation scheme with an electron beam travelling over a finitely thick slab made of negative-index materials. Such a scheme could start oscillating from Cherenkov radiation without external reflectors, since the electromagnetic energy flows backward inside the negative-index materials, leading to inherent feedback. We theoretically analyzed the mechanism of beam-wave interaction, and designed the negative-index material.**

I. INTRODUCTION

CURRENTLY, there is a surge of interest on the research of negative-index material (NIM), which shows many exotic and remarkable electromagnetic phenomena, such as reversed Cherenkov radiation and reversed Doppler shifts. Recent successes in fabricating these artificial materials [1-4] have initiated an exploration into the use of them to investigate new physics and to develop new applications.

As is known, electromagnetic surface wave along the interface between the vacuum and a certain material play an important role in development of a Cherenkov type radiation source. An electron beam travelling along the surface can interact with the electromagnetic surface wave with phase velocity synchronizing the electron's velocity and amplify the wave when oscillation conditions are satisfied. We find that the electromagnetic waves on the surface of a media made of negative-index materials hold special features. With using these features a novel radiation scheme is presented in this paper. We also designed the unit cell of negative-index material with the help of FDTD simulation.

II. THEORY

We consider a two-dimensional Cartesian coordinate. The electrons with velocity v_0 initially move in the z direction in vacuum along the trajectories $a < x < a+d$ over a finitely thick slab with a perfect conductor substrate, and the electrons are coupled with the transverse magnetic (TM) mode of the electromagnetic wave. The system is assumed to be immersed in a strong external magnetic field such that the electrons move only in the z direction. Using boundary conditions, the dispersion relation between frequency ω and axial wave number k for the waveguide mode can be directly derived as

$$\varepsilon_r \frac{\alpha}{\tau} = -G\tanh(\tau H) \tag{1}$$

where

$$G = \frac{1+g}{1-g},$$

$$g = \frac{(\varepsilon_g - 1)(1-e^{-2d\alpha\sqrt{\varepsilon_g}})e^{-2a\alpha}}{(\sqrt{\varepsilon_g}+1)^2 - (\sqrt{\varepsilon_g}-1)^2 e^{-2d\alpha\sqrt{\varepsilon_g}}},$$

$$\varepsilon_g = 1 - \frac{\omega_b^2}{\gamma^3(\omega - kv_0)^2},$$

$$\alpha^2 = k^2 - \frac{\omega^2}{c^2},$$

$$\tau^2 = k^2 - \frac{\omega^2 \varepsilon_r \mu_r}{c^2},$$

and ω_b is the beam plasma frequency. Considering the operating point (ω_0, k_0) and using the small signal theory, we worked out the spatial growth rate of amplitude,

$$\mu = \mathrm{Im}(\delta k) = \frac{\sqrt{3}}{2}\left| -\frac{\frac{1}{2}(e^{-2\alpha d}-1)e^{-2\alpha a}\tanh(\tau H)\omega_b^2}{\gamma^3\beta^2c^2\xi'(\omega_0,k_0)}\right|^{\frac{1}{3}} \tag{2}$$

where

$$\xi' = \frac{\varepsilon_r k}{\alpha\tau} - \frac{\varepsilon_r \alpha k}{\tau^{3/2}} + \frac{(1-\tanh(\tau H)^2)kH}{\tau}.$$

The power-flow above and below the interface along the z direction is respectively expressed as

$$S_a = \int_0^\infty dx \frac{1}{2}E_x H_y^* = \frac{kc^2|C|^2}{4\mu_0\alpha\omega} \tag{3}$$

$$S_b = \int_{-H}^0 dx \frac{1}{2}E_x H_y^* = \frac{kc^2|C|^2(e^{4\tau H} + 4e^{2\tau H}H\tau - 1)}{4\mu_0\omega\varepsilon_r\tau(e^{2\tau H}+1)^2} \tag{4}$$

where C is the coefficient to be determined. From these two expressions we understand that the energy above and below the interface can flow in opposite directions when the slab is made from negative-index material ($\varepsilon_r < 0$, $\mu_r < 0$), and this effect leads to inherent feedback. The wave above the interface interacts with the electron beam moving to the z direction and the whole waves are amplified during the interaction; at the downstream end, the wave above the interface goes out the slab (they are partially reflected and partially diffracted there; and hereafter we ignore the reflections in the present theory); while the energy carried by the wave below the interface are retained. Note that they are retained but not reflected, since it intrinsically move to the –z direction; energy carried by the wave below the interface is reapportioned among the whole waves to satisfy the boundary condition on the surface of a slab; and at the upstream end the energy carried by the wave above the interface is retained, and start the second round trip. Thus, the system is possible to start oscillating even without external reflectors.

We define the total power-flow as $S_{total} = |S_a| + |S_b|$, and the ratios for the power-flow above and below the interface are written as $\rho_a = |S_a|/S_{total}$,and $\rho_b = |S_b|/S_{total}$, respectively. The condition to start oscillating should be

$$e^{2\mu L}\rho_a\rho_b = 1 , \qquad (5)$$

where L is the total length of a slab.

The slab is assumed to be made from a homogeneous, isotropic, linear negative-index material, and the Drude model is used to express the permeability and permittivity, $\varepsilon_r(\omega) = \varepsilon_\infty - \omega_p/\omega(\omega - i\nu_c)$, where ω_p is the plasma frequency and ν_c is the collision frequency. For the simplicity of numerical calculation and computer simulation, we assume $\varepsilon_r = \mu_r$. As an example, we choose the parameters as $\varepsilon_\infty = 1$, $\omega_p = 32\pi\times10^9$ rad/s, $\nu_c = 10^{-4}$ Hz. The permeability and permittivity are negative below the plasma frequency. The geometrical parameters used in this paper are H=2 mm, d=1 mm, a=0.5 mm, and the length of the plate is L=4 cm. When a rectangular beam with width same to the width of the slab, $w = 4$ mm, is assumed, the start current can be worked out. We performed computer simulation of a continuous electron beam with PIC code. The beam current is set to be 3 A, and the initial electron energy is 100 keV. The other parameters are same to those mentioned above. The temporal behavior of y-directed magnetic field is observed very close to the surface of the slab, and it is shown in Fig. 1. From Fig.1 we know that the device can oscillate without using external reflectors.

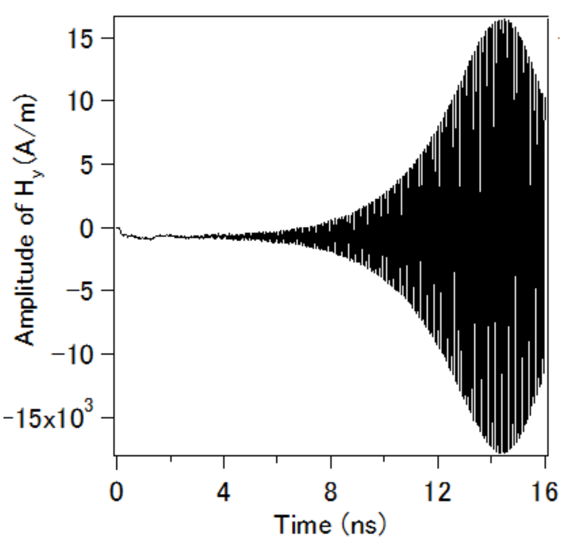

Fig.1 Time signal of Hy of electromagnetic wave to show the oscillation of a Cherenkov radiation oscillator without external reflectors.

III. UNIT CELL DESIGN

We present the design of an isotropic negative-index metamaterial. The material consists of split rings and wires in each of three orthogonal planes, in order to respond to the full electromagnetic field components. The 3D structure of the split rings and wires for the unit cell is as shown in Fig. 2(a). The unit cell is cubic with a size of about 2mm on a side. The copper is adopted to fabricate the ring and wire. With the help of FDTD simulations, we demonstrated that such a structure shows negative permeability and permittivity at about 13 GHz, as is shown in Fig.2 (b).

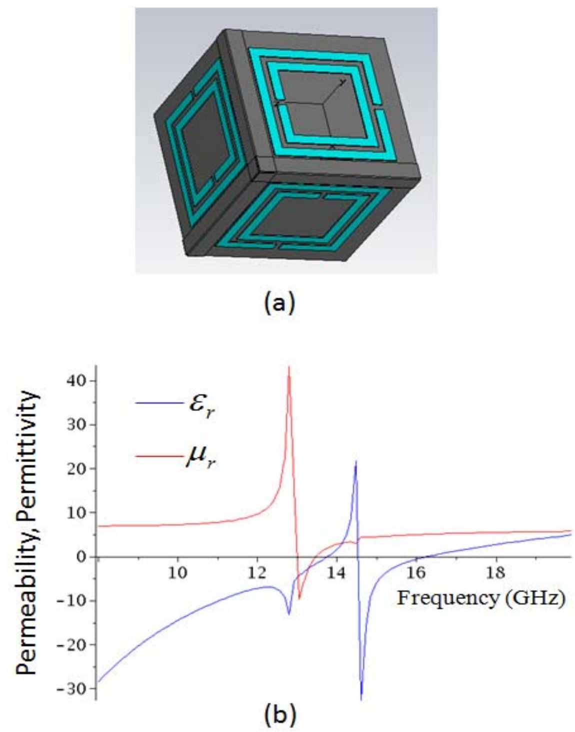

Fig.2 (a) Unit cell of negative-index material and (b) corresponding permeability and permittivity

IV. SUMMARY

The electromagnetic wave radiation from the interaction between an electron beam and negative-index material was studied. Based on the characteristics of the electromagnetic modes, a novel radiation source was proposed and demonstrated with theoretical analysis and simulations. This work is partially supported by KAKENHI (No. 24560057), Research Foundation for Opto-Science and Technology, and Matsuo Research Foundation.

REFERENCES

[1]. R.A. Shelby, D.R. Smith, S. Schultz, “Experimental verification of a negative index of refraction,”Science 292, 77-79, 2001

[2]. H.-T Chen, W.J. Padilla, J.M.O. Zide, A.C. Gossard, A.J. Taylor and R. D. Averitt,” Active terahertz metamaterial devices,” Nature **444**,597-600,2006

[3]. D. R. Smith,W. Padilla, D. C. Vier, S.C. Nemat-Nasser,and S. Schultz, “Composite medium with simultaneously negative permeability and permittivity” Phys. Rev. Lett. 84, 4184-4187, 2000.

[4]. S. Foteinopoulou, E. N. Economou, and C. M. Soukoulis, “Refraction in media with a negative refractive index,” Phys. Rev. Lett. 90, 107402 , 2003