Plasmonic superfocusing of THz waves in metallic V-groove tapered waveguide theoretically considered by quasi-separation of variables

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***Abstract*—A non-adiabatic superfocusing theory of THz waves in a metallic V-groove tapered waveguide is studied by quasi-separation of variables for the explanation of our recent experimental results (*Appl. Phys. Express,* vol. 7, 112401, 2014). Approximate analytical solutions are obtained for the two regions at the apex and in the vicinity of it, where as the result the property of plasmonic superfocusing is analytically discussable in details.**

I. INTRODUCTION

PASMONIC waveguides of a tapered structure such as V-groove [1] and cone [2] have the capability to confine terahertz (THz) electromagnetic waves into an area smaller than the diffraction limit. The concept of plasmonic superfocusing in the tapered waveguide was introduced by the research in optical region [3] and developed as adiabatic superfocusing in optical [4] and THz [5] regions. Experimental results recently obtained in our group [1] for the metallic V-groove tapered waveguide cannot be understood with the concept of adiabatic superfocusing in THz region [5] because of the large taper angle; therefore, a new concept of non-adiabatic superfocusing in THz region is requisite for the explanation of our experimental results. This paper describes an analytical theory of non-adiabatic superfocusing in metallic V-groove structures for THz region based on quasi-separation of variables [6], which is already applied to those in optical region.

II. RESULTS

According to the nonadiabatic superfocusing theory [6] for the metallic V-groove structure as shown in Fig.1, the wave equation of the transverse magnetic wave is solved by assuming solutions of quasi-separation of variables, $H_z(\rho,\phi)=R(\rho)Q(\phi,\rho)$, with perturbation methods. A solvable set of the zeroth-order equations are given by

$$\frac{\partial^2}{\partial\rho^2}R_u^{(0)}(\rho)+\frac{1}{\rho}\frac{\partial}{\partial\rho}R_u^{(0)}(\rho)+\left(k_p^2+\frac{\zeta_u^{(0)}(\rho)}{\rho^2}\right)R_u^{(0)}(\rho)=0 \qquad (1)$$

$$\frac{\partial^2 Q^{(0)}(\phi,\rho)}{\partial\phi^2}-\left\{\eta_j(\rho)\right\}^2 Q^{(0)}(\phi,\rho)=0 \qquad (2)$$

with

$$\eta_j(\rho)=\sqrt{\beta_j^2\rho^2+\zeta_u^{(0)}(\rho)},\quad \beta_j=\sqrt{k_p^2-\varepsilon_j k_0^2},\quad j=1,2 \qquad (3)$$

where k_p is the wavenumber of planar surface plasmon polaritons given by $k_p=k_0\sqrt{\varepsilon_1\varepsilon_2/(\varepsilon_1+\varepsilon_2)}$ and k_0 is the wavenumber of light in vacuum given by $k_0=2\pi/\lambda_0(=\omega/c)$. From continuity of the metal-dielectric surface ($\phi=\alpha$) for the radial electric field, we obtain the boundary condition for even

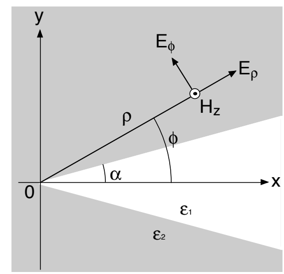

Fig. 1. Geometry for a metallic V-groove structure in the cylindrical coordinates. ε_1 and ε_2 are the dielectric and metallic permittivities, respectively, inside and outside the V-groove.

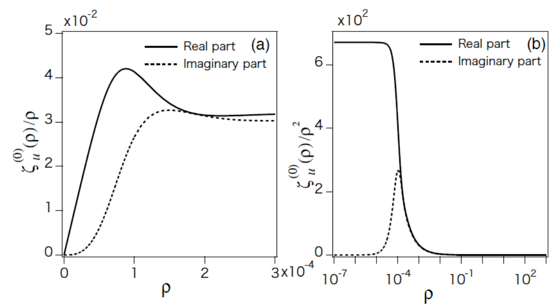

Fig. 2. Numerical calculations of (a) $\zeta_u^{(0)}(\rho)/\rho$ and (b) $\zeta_u^{(0)}(\rho)/\rho^2$ as a function of ρ using $\varepsilon_1=1$, $\varepsilon_2=-3.2\times10^4+i6.7\times10^5$, $2\alpha=20^\circ$, $\lambda_0=1$ in Eq. (4).

symmetry given by which can determine the quasi-separation quantity, $\zeta_u^{(0)}(\rho)$.

$$\frac{\tanh\{\alpha\eta_1(\rho)\}}{\tanh\{(\pi-\alpha)\eta_2(\rho)\}}+\frac{\varepsilon_1\eta_2(\rho)}{\varepsilon_2\eta_1(\rho)}=0 \qquad (4)$$

By numerically solving Eq. (4), we observed that $\zeta_u^{(0)}(\rho)/\rho$ and $\zeta_u^{(0)}(\rho)/\rho^2$ were constant for some regions of ρ as shown in Fig. 2. It is then found that the boundary condition in Eq. (4) is approximately solved as

$$\frac{\zeta_u^{(0)}(\rho)}{\rho}=-\frac{1}{\alpha}\frac{\varepsilon_1}{\varepsilon_2}\sqrt{\beta_2^2}\sim\frac{k_0\varepsilon_1}{\alpha\sqrt{|\varepsilon_2|}}\exp\left\{i\left(\frac{\pi}{4}-\frac{1}{2}\arg\varepsilon_2\right)\right\} \qquad (5)$$

for the rage of $1/2\pi\alpha\sqrt{|\varepsilon_2|}\ll\rho/\lambda_0\ll\sqrt{|\varepsilon_2|}/2\pi\alpha\varepsilon_1$, and

$$\zeta_u^{(0)}(\rho)/\rho^2\sim(\pi/\alpha-1)k_0^2 \qquad (6)$$

for the rage of $0\le\rho/\lambda_0\ll1/2\pi^2\sqrt{|\varepsilon_2|}$, where λ_0 is the wavelength of light in free space. These analytical solutions of $\zeta_u^{(0)}(\rho)$ allow us to discuss detailed properties of THz superfocusing in the metallic groove tapered waveguide.

For THz superfocusing at the apex for the rage of

$0 \le \rho/\lambda_0 \ll 1/2\pi^2\sqrt{|\varepsilon_2|}$, by substituting Eq. (6) into Eq. (1), we obtain

$$\frac{\partial^2}{\partial\rho^2}R_u^{(0)}(\rho)+\frac{1}{\rho}\frac{\partial}{\partial\rho}R_u^{(0)}(\rho)+\frac{\pi}{\alpha}\mathrm{k}_0^2R_u^{(0)}(\rho)=0, \qquad (7)$$

which easily gives a solution of $R_u^{(0)}(\rho)\sim J_0(\rho k_0\sqrt{\pi/\alpha})$, where J_0 is the Bessel function of the first kind with an order of 0. The solution shows that wavenumber of THz superfocusing at the apex is given by $k_0\sqrt{\pi/\alpha}$; it means that the wavenumber at the apex is increased by a factor of $\sqrt{\pi/\alpha}$ as compared with the free-space wavenumber, k_0, and that the wavenumber increases with a decrease in taper angle, 2α. These behaviors of THz superfocusing are clearly different from those of optical superfocusing considered in [6].

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