Self-consistent Modeling of Terahertz Waveguide and Cavity with Frequency-dependent Conductivity

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***Abstract*—Ohmic dissipation can lead to excessive wall losses at terahertz (THz) frequencies, while the high-frequency oscillatory motion of conduction electrons tends to mitigate the collisional damping. In this study, a frequency-dependent conductivity is used to model the wall losses on the waveguides and open cavities commonly employed as gyrotron interaction structures. The reduction in Ohmic losses under the AC-conductivity model is shown to be increasingly significant as the frequency reaches deeper into the THz region. Such effects are of considerable importance to THz gyrotrons for which the minimization of Ohmic losses constitutes a major design consideration.**

I. INTRODUCTION

METAL is a key part of most high-power waveguide structures. In an AC electric field with exp($-i\omega t$) time dependence, it is characterized by a complex AC conductivity given by the Drude model

$$\sigma_{AC} = \frac{\sigma_{DC}}{1-i\omega/\gamma_0}, \quad (1)$$

where γ_0 is the free-electron collision frequency and σ_{DC} is the DC conductivity. In Eq. (1), as the value of ω crosses from well below to well above γ_0, σ_{AC} turns from a predominantly real to a predominantly imaginary number. Thus, the ratio of ω to γ_0 determines the nature of the wave behavior in a conducting medium over the vast span of the electromagnetic spectrum. A typical value of γ_0 is of the order of $10^{13\text{-}14}$/sec. At room temperature, for example, γ_0 (copper) $\approx 4\times10^{13}$/sec and γ_0 (silver) $\approx 2\times10^{13}$/sec. In the low-frequency limit, $\omega \ll \gamma_0$ (up to microwave frequencies), we have $\sigma_{AC} \approx \sigma_{DC}$ (a real quantity). This is the good-conductor regime characterized by a shallow field penetration depth (skin depth).

In the high-frequency limit, $\omega >> \gamma_0$ (near infrared and beyond), σ_{AC} is dominated by the imaginary part. In this limit, the free electron motion repeatedly reverses directions within one mean free path, which greatly mitigates the collision effects. As a result, the conductor no longer behaves as a lossy medium. It is instead characterized by the evanescence of visible light and transparency to frequencies above the ultraviolet.

The THz region (nominally 10^{11}-10^{13} Hz) is a marginal case. The lower portion of the region falls in the $\omega \ll \gamma_0$ regime, where σ_{DC} adequately characterizes the conducting medium. However, for the upper portion (where $\omega \geq \gamma_0$), the frequency-dependent term in Eq. (1) can no longer be neglected, which calls into question of the validity of the DC conductivity over the full THz range.

II. RESULTS

First-order dispersion relations of a circular waveguide are derived, using a two-medium formalism to incorporate the AC conductivity and self-consistent fields. Dispersive properties and attenuation constants at frequencies under and above the cutoff are numerically illustrated and interpreted [1].

Let $\alpha(\sigma_{AC})$ and $\alpha(\sigma_{DC})$ be the attenuation constants evaluated under the AC- and DC-conductivity models, respectively. The deviation between $\alpha(\sigma_{AC})$ and $\alpha(\sigma_{DC})$ is displayed in Fig. 1 by their ratio plotted over a broad frequency range for the TE_{04} mode in copper and silver waveguides. For a meaningful comparison, the waveguide radius (r_w, upper scale in Fig. 1) is varied and, for each r_w, f is set at 1.06 times the cutoff frequency (f_c) as is typical of the gyrotron operating frequency relative to f_c. The $\alpha(\sigma_{AC})$ to $\alpha(\sigma_{DC})$ ratio shows a steep drop at frequencies above 1 THz. The same formalism is extended to model the open cavity shown in Fig. 2. Figure 3 plots the ratio of $Q_{ohm}(\sigma_{AC})$ to $Q_{ohm}(\sigma_{DC})$ as a function of the mode frequency. Again, the Ohmic loss under the AC-conductivity model is much reduced from that of the DC-conductivity model at the high end of THz frequencies.

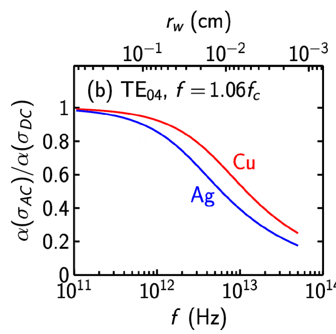

Fig. 1.Ratio of $\alpha(\sigma_{AC})$ to $\alpha(\sigma_{DC})$ as a function of wave frequency f for the TE_{04} mode in copper (Cu) and silver (Ag) waveguides.

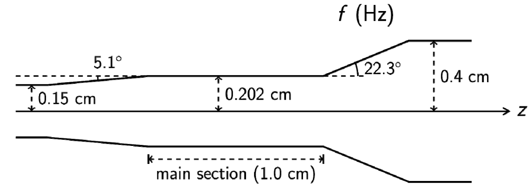

Fig. 2. The open cavity under study

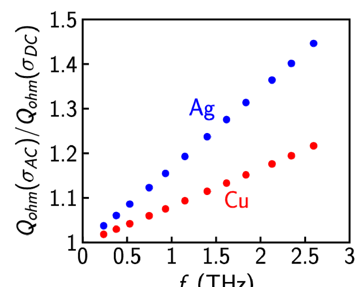

Fig. 3.Ratio of Q_{ohm} (σ_{AC}) to Q_{ohm} (σ_{DC}) as a function of the mode frequency.

REFERENCES

[1] Y. J. Huang, K. R. Chu, and M. Thumm, Phys. Plasmas **22**, 013108 (2015).