# Algebraic Reconstruction Technique for Millimeter-wave Holographic Imaging

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*Abstract*—**Millimeter-wave (MMW) holographic imaging has been intensively investigated for the application of personnel inspection. In order to obtain high resolution images, synthetic aperture technique can be used to form large virtual aperture. However, the effective size of the synthetic aperture is always limited by the beamwidth of the transmitting and receiving antennas. In this paper, algebraic reconstruction technique (ART) is introduced for the reconstruction of MMW images with consideration of the radiation pattern of antennas. Since the effect of the radiation pattern is compensated, high resolution images can be achieved by overcoming the resolution limitation which is determined by the beamwidth of antennas. Simulations are presented to verify the proposed ART based MMW imaging reconstruction method.**

## I. INTRODUCTION

MILLIMETER-WAVE (MMW) holographic imaging holds large potential in the application of personnel inspection since that MMW is nonionizing, can penetrate the clothing barrier, and has a good resolution capability. Compared with X-ray backscatter imaging technique, the major advantage of MMW imaging is the security feature. However, the imaging resolution still need to be improved due to the relative long wavelength.

There are mainly two ways to improve the resolution of the imaging results by MMW holographic imaging. One way is to adopt the higher frequency source, for example the frequency approaching terahertz band[1]. This can be achieved with the development of the hardware though the performance of functional devices for submillimeter wave is now rather poor. Another way is utilizing the advanced imaging approach based on the existing mature systems. Synthetic aperture technique which is widely used in radar imaging has been extended to MMW holographic imaging. The core idea of synthetic aperture is taking advantage of the movement of antennas to obtain the backscatter data from wide angles. Therefore, wide beamwidth is essential for the synthetic aperture imaging of near-field MMW imaging. Sheen et al.[2] used wide-beamwidth 10-40 GHz antennas to achieve high resolution MMW images which are as good as the results obtained by 75-105 GHz narrow-beamwidth antennas. Note that the imaging reconstruction algorithm used here assumes that the radiation pattern is a completely flat function along the 180-degree angle.

In this paper, algebraic reconstruction technique (ART)[3] is introduced to reconstruct high resolution MMW images by compensating the radiation pattern of antennas. In ART reconstruction, the discrete coefficient matrix can give an accurate description of any imaging process. Hence the radiation pattern of antennas and arbitrary imaging geometry can be taken into account. The disadvantage of ART reconstruction is the high computation cost which should be concerned in practical applications.

## II. METHOD

In planar MMW holographic imaging, the transmitting and receiving antennas are bundled together to move along a rectangle aperture. During the scan, the target reflects the MMWs and the response of the receiving antenna can be regarded as the superposition of the reflected signals from all directions. Due to the heterodyne mixing technique, the round-trip phase decay can be easily extracted without using any reference wave. Under the Born approximation and the assumption of isotropic scattering, the response of the receiving antenna can be described as:

$$s(x_0,y_0)=\iint f(x,y)\frac{\exp(-j2kr)}{r^2}\,\mathrm{d}x\,\mathrm{d}y\,, \tag{1}$$

where $(x_0,y_0)$ is the coordinate of the antennas, $f(x,y)$ is the complex-valued reflectivity of the target, $k$ is the wavenumber of the adopted source, $r=\left[(x-x_0)^2+(y-y_0)^2+Z_0^2\right]^{1/2}$ and $Z_0$ is the imaging distance. Based on scalar diffraction theory, we have proposed an exact reconstruction formula to recover $f(x,y)$ [4]:

$$f(x,y)=\left[\frac{2Z_0}{j\lambda}\mathrm{FT}_{2D}^{-1}\left[\mathrm{FT}_{2D}\left[s^*(x_0,y_0)\right]\exp\left(-jZ_0\sqrt{4k^2-k_x^2-k_y^2}\right)\right]\right]^*, \tag{2}$$

where $k_x=2\pi f_x$, $k_y=2\pi f_y$ are the spatial components of the wave vector $\mathbf{k}$, and $\mathrm{FT}_{2D}$ and $\mathrm{FT}_{2D}^{-1}$ represent the 2D Fourier transform and its inverse, respectively.

The above formula can give an exact numerical estimate of the complex-valued reflectivity of the target if the responses of antennas are uniform for all directions. However, the beamwidths of antennas are always limited in real imaging cases, especially for the antennas working at frequencies toward terahertz frequency. When the beamwidth of the antenna is narrow, the imaging resolution will be seriously restricted and increasing the scanned aperture will be no use.

An ART based MMW reconstruction method is presented here to improve the imaging resolution by considering the radiation pattern of antennas. In ART reconstruction, the imaging process can be exactly depicted by using the discrete coefficient matrix. Following the imaging model described by formula (1), the responses of antennas with radiation pattern $H(\theta)$ will be:

$$s(x_0,y_0)=\iint f(x,y)\frac{\exp(-j2kr)}{r^2}H^2(\theta)\,\mathrm{d}x\,\mathrm{d}y\,. \tag{3}$$

Then the element of the coefficient matrix can be expressed as:

$$A_{(x,y,x_0,y_0)}=\frac{\exp(-j2kr)}{r^2}H^2(\theta)\Delta x\Delta y\,, \tag{4}$$

and the MMW imaging process can be written in the matrix form:

$$\mathbf{A}\mathbf{x} = \mathbf{b}\,, \tag{5}$$

where $\mathbf{A}$ is the coefficient matrix, $\mathbf{x}$ and $\mathbf{b}$ are the vectors of the complex-valued reflectivity and the sampled holographic data. As for the complex-valued reconstruction problem, the real and imaginary parts can be separated:

$$\begin{bmatrix} \mathbf{A}^R & -\mathbf{A}^I \\ \mathbf{A}^I & \mathbf{A}^R \end{bmatrix} \begin{bmatrix} \mathbf{x}^R \\ \mathbf{x}^I \end{bmatrix} = \begin{bmatrix} \mathbf{b}^R \\ \mathbf{b}^I \end{bmatrix}. \tag{6}$$

Therefore, we can get two linear equations:

$$\begin{aligned} \mathbf{A}^R\mathbf{x}^R - \mathbf{A}^I\mathbf{x}^I &= \mathbf{b}^R \\ \mathbf{A}^I\mathbf{x}^R + \mathbf{A}^R\mathbf{x}^I &= \mathbf{b}^I \end{aligned}\,. \tag{6}$$

Finally, the ART based MMW reconstruction method can be obtained:

$$\mathbf{x}_1^R = \mathbf{x}_{iter}^R - \frac{\mathbf{b}^R - \mathbf{a}_i^R\mathbf{x}_{iter}^R + \mathbf{a}_i^I\mathbf{x}_{iter}^I}{\mathbf{a}_i^R(\mathbf{a}_i^R)^T + \mathbf{a}_i^I(\mathbf{a}_i^I)^T}(\mathbf{a}_i^R)^T \qquad \mathbf{x}_1^I = \mathbf{x}_{iter}^I - \frac{\mathbf{b}^R - \mathbf{a}_i^R\mathbf{x}_{iter}^R + \mathbf{a}_i^I\mathbf{x}_{iter}^I}{\mathbf{a}_i^R(\mathbf{a}_i^R)^T + \mathbf{a}_i^I(\mathbf{a}_i^I)^T}(\mathbf{a}_i^I)^T$$

$$\mathbf{x}_2^R = \mathbf{x}_1^R + \frac{\mathbf{b}^I - \mathbf{a}_i^I\mathbf{x}_1^R - \mathbf{a}_i^R\mathbf{x}_1^I}{\mathbf{a}_i^R(\mathbf{a}_i^R)^T + \mathbf{a}_i^I(\mathbf{a}_i^I)^T}(\mathbf{a}_i^R)^T \qquad \mathbf{x}_2^I = \mathbf{x}_1^I + \frac{\mathbf{b}^I - \mathbf{a}_i^I\mathbf{x}_1^R - \mathbf{a}_i^R\mathbf{x}_1^I}{\mathbf{a}_i^R(\mathbf{a}_i^R)^T + \mathbf{a}_i^I(\mathbf{a}_i^I)^T}(\mathbf{a}_i^I)^T$$

$$\mathbf{x}_{iter+1} = \mathbf{x}_2^R + j\mathbf{x}_2^I$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are intermediate variables.

## III. RESULTS

Simulations are performed to validate the effectiveness of the proposed ART based MMW reconstruction algorithm. We construct a line-pair model as the imaging target, as shown in Fig. 1a. The size of the target is $10\times10$ cm with $400\times400$ pixels. The rectangles are regarded as metal material with the reflectivity of 1. The spatial resolutions of the line pairs are 2 LP/mm, 1 LP/mm, 1/2 LP/mm, 1/4 LP/mm and 1/8 LP/mm from top to bottom, respectively. The frequency of the source is set to be 300 GHz. The holographic data are sampled along a square aperture of which the position and the size are the same as that of the target. Note that the sampled interval can be set as half of the wavelength (0.5 mm).Then the number of the sampled data will be $200\times200$.The imaging distance here is set to be10 cm.

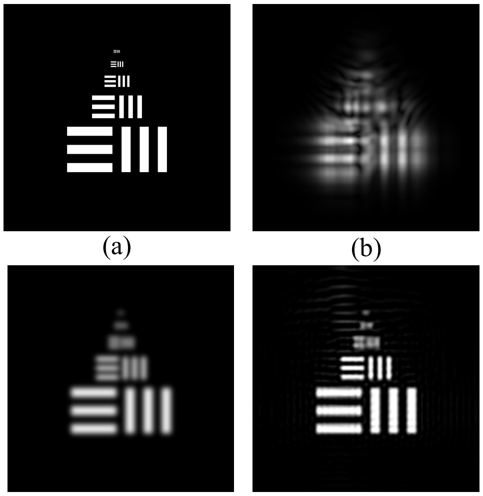
(a) (b)
(c) (d)

**Fig. 1.** Line-pair model (a), magnitude of the holographic data (b), reconstructed result by FFT based exact reconstruction (c), reconstructed result by ART reconstruction (d).

In order to be closer to the actual imaging configuration, the diagonal horn WR 2.8 (the working frequency region is 260-400 GHz) made by Virginia Diodes Inc. forms the transmitting and receiving antennas. Figure 2 shows the optical image of the diagonal horn. The radiation pattern of diagonal horn can be calculated by Love's method[5] and the result for WR 2.8 diagonal horn working at 300 GHz is presented in Fig. 3. The calculated full 3 dB beamwidth is 12.7 degree. Consequently, the narrow beamwidth will obviously blur the reconstructed MMW image by the exact FFT based method.

**Fig.2.** Optical image of the WR 2.8 diagonal horn.

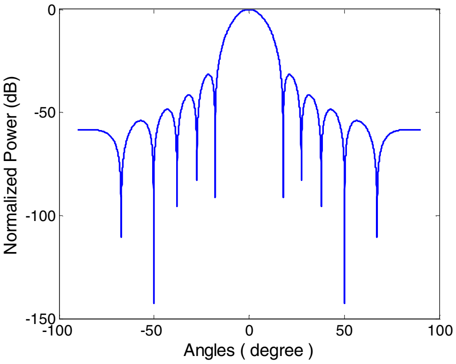

**Fig.3.** Radiation pattern of the WR 2.8 diagonal horn.

The simulated holographic data can be calculated by discretizing formula (3). Figure 1b presents the magnitude image of the holographic data which is obscure due to the defocusing effect. Fig. 1c shows the reconstructed image (real part) by the exact FFT based reconstruction. Though the calculation is fast, the images are seriously blurred due to the narrow beamwidth. In contrast, obvious resolution improvement can be observed from the reconstructed result by ART based algorithm, which are shown in Fig. 1d (the number of the iteration is 40).

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