Feasibility Study of TM Modes for Electron Cyclotron Maser

Tsun-Hsu Chang, Wei-Chen Huang, and Wei-Chen Chen
Department of Physics, National Tsing Hua University, HsinChu, 30013 Taiwan

***Abstract*—The transverse magnetic (TM) modes have long been considered as the unsuitable waveguide modes for the operation of the electron cyclotron maser (ECM). This study investigates the linear behavior of the TM modes and reveals for the first time that certain TM modes might be suitable for gyrotrons --- ECM based devices. In addition, non-linear but non-self-consistent model shows that for a fixed field profile the efficiency could be as high as that of the transverse electric (TE) modes. Such interesting findings deserve more theoretical and experimental studies, and might facilitate some applications.**

I. INTRODUCTION

THE transverse magnetic (TM) modes are generally excluded from the operating modes of the electron cyclotron masers for three reasons. First, the longitudinal electric field component E_z will cause the axial bunching, which will compete with the azimuthal bunching, resulting in a much lower efficiency. Second, according to the relativistic transformation of electromagnetic fields, the transverse electric field components might disappear when choosing a specific reference frame, resulting in unsuitable bunching. Third, the E_z component will modulate the axial velocity v_z, which will cause significant velocity spread and then reduce the efficiency. The three arguments are strong enough, so there has been little research on this topic [1-5]. Interesting, those studies don't complete rule out the possibility of using TM mode as the operating mode.

Here, we develop a linear theory and a non-self-consistent nonlinear model. Results show that for the mode with azimuthal symmetry, such as TM_{mn} (m=0), traditional understanding is correct, that is TM mode is not suitable for gyrotrons. However, some TM modes are as good as TE modes and have unique properties.

II. RESULTS

The electromagnetic fields of the right circularly polarized TM mode is:

$$E_z = \mathrm{Re}\left[k_{mn}^2 E_0 J_m(k_{mn}r)e^{-i(\omega t - m\theta - k_z z)}\right]$$

$$E_r = \mathrm{Re}\left[k_{mn} i k_z E_0 J_m'(k_{mn}r)e^{-i(\omega t - m\theta - k_z z)}\right]$$

$$E_\theta = \mathrm{Re}\left[-\frac{m}{r}k_z E_0 J_m(k_{mn}r)e^{-i(\omega t - m\theta - k_z z)}\right]$$

$$B_r = \mathrm{Re}\left[\frac{\omega m}{cr}E_0 J_m(k_{mn}r)e^{-i(\omega t - m\theta - k_z z)}\right]$$

$$B_\theta = \mathrm{Re}\left[\frac{i\omega k_{mn}}{c}E_0 J_m'(k_{mn}r)e^{-i(\omega t - m\theta - k_z z)}\right]$$

$$B_z = 0$$

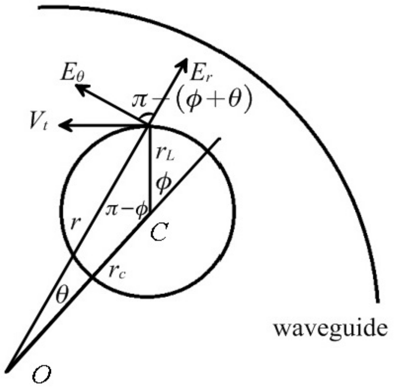

Fig. 1. Model of cyclotron interactions. E_θ and E_r are the electric fields of the TM_{mn} mode of a circular waveguide. Point O is the center of the waveguide. Point C is the center of electron gyration (guiding center).

Starting from the energy and Lorentz force equations, we have

$$\begin{aligned}\eta &= \frac{-1}{\gamma_0 - 1}\int_0^t \left\langle \frac{d}{dt}\gamma \right\rangle_{\phi_0''} dt \\ &= \frac{e^2\omega}{4\gamma_0(\gamma_0-1)m_e^2c^4}\left(-s\frac{k_z}{k_c^2 r_L}v_{\perp 0}v_{z0} + v_{z0}^2\right)\left(1-\frac{c^2}{\omega v_{z0}}k_z\right)C_{sm} \\ &\times E_{0z}^2\left(-\frac{t^2}{\varepsilon}\cos\varepsilon t + \frac{2t}{\varepsilon^2}\sin\varepsilon t + \frac{2}{\varepsilon^3}\cos\varepsilon t - \frac{2}{\varepsilon^3}\right)\end{aligned}$$

where $C_{sm} = J_{s-m}^2(k_c r_c)J_s^2(k_c r_L)$ is the TM mode's coupling coefficient, $\varepsilon = \omega - k_z v_{z0} - s\Omega_e/\gamma$, and other symbols are the same as those of TE mode.

In comparison with TE mode, the linear theory of the efficiency is:

$$\begin{aligned}\eta &= \frac{e^2\omega}{4\gamma_0(\gamma_0-1)m_e^2c^4}(v_{\perp 0}^2)\left(1-\frac{k_z^2c^2}{\omega^2}\right)H_{sm} \\ &\times \hat{E}^2\left(-\frac{t^2}{\varepsilon}\cos\varepsilon t + \frac{2t}{\varepsilon^2}\sin\varepsilon t + \frac{2}{\varepsilon^3}\cos\varepsilon t - \frac{2}{\varepsilon^3}\right),\end{aligned}$$

where $H_{sm} = J_{s-m}^2(k_c r_c)J_s'^2(k_c r_L)$ is the TE mode's coupling coefficient, and other symbols are the same as those of TM mode.

Three are three major differences between TE and TM modes. First, the efficiency of TM mode depends on v_z, but TE mode is not. Second, the coupling coefficient of TE and TM modes are different. They are C_{sm} and H_{sm}, respectively. Third, TM mode depends on the propagating constant k_z but

TE mode depends on k_z^2. The third properties is unique and the characteristics are shown in Fig. 2.

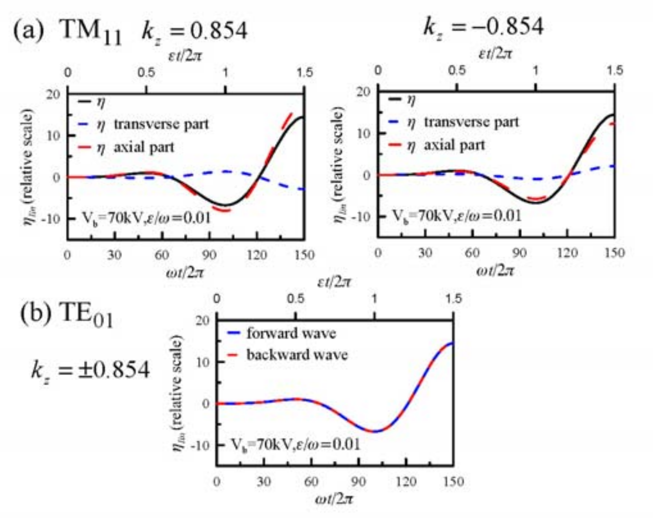

Fig. 2. The change of efficient η_{lin} at linear stage $\omega t/2\pi$ for (a) TM$_{11}$ mode and (b) TE$_{01}$ mode. TM$_{11}$ mode depends on sign of the propagating constant, while TE$_{01}$ mode doesn't.

III. SUMMARY

This study shows that some TM modes may be suitable for the gyrotron operation. The difference characteristics in forward wave and backward waves might be beneficial for the backward wave oscillator. The TM modes deserve more theoretical and experimental studies.

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