Refractivity of Water Vapor at Terahertz Frequencies – Comparison of Measurements with Models

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***Abstract*—Refractivity of water vapor at THz frequencies was calculated using three different models. These did not agree. A direct measurement was made using time-domain spectroscopy and the correct form of the refractivity model was determined.**

I. INTRODUCTION

ASTRONOMICAL observations at millimeter and sub-millimeter wavelengths (frequencies of 0.1 to 1THz) are affected by both absorption and refraction in the atmosphere. To first order, the absorption simply reduces the strength of the signals and increases the noise (because of the associated emission) while the refraction causes a shift in the apparent position of the source. To obtain angular resolution of one arc second or better at these wavelengths it is however necessary to use the aperture synthesis technique, where the signals from a number of separate dishes are combined to form the image. In this case the signals travel along different paths through the atmosphere to the individual dishes so differential effects can become important. Changes in the atmospheric conditions cause fluctuations in the effective path length which will alter the phase of the signals, resulting in reduced coherence and degraded images. Water molecules have a large dipole moment so they make a major contribution to the refractive index of air at THz frequencies, even though water is a minor constituent. As a result fluctuations in the water vapor content along the line of sight typically produce a large part of the path variations seen at these wavelengths.

At ALMA[1], a 66-element aperture synthesis telescope in Chile, radiometers mounted on each of the dishes are used to observe the emission from the water along the line of sight at frequencies close to 183 GHz, where there is a spectral line due to the $3_{1,3}$ to $2_{2,0}$ rotational transition. There is a close relationship between the emission and the path changes, so these measurements can be used to make a correction for the phase changes[2]. To do this accurately, however, it is necessary to have a reliable model for the refractive index of water vapor as a function of frequency and in particular this model needs to take proper account of the dispersive effects.

Several different models of atmospheric refraction are in use in the radio astronomy community. Tests at ALMA suggested that the model being used was exaggerating the dispersive effects by roughly a factor of two. A check with a second, independent, software package was therefore made. To our surprise this gave a similar result, i.e. discrepant with the data. A third model was however found to give results in agreement with the measurements. Since the tests at the telescope were difficult, it was decided to clarify the position by carrying out a measurement in a controlled laboratory environment using a THz time-domain spectrometer (TDS).

II. RESULTS

The THz TDS used for measuring the refractivity of water vapor in the atmosphere had a standard configuration, consisting of a biased photoconductive emitter, an electro-optic detector, and four parabolic mirrors. The length of the THz beam path was 104 cm. Dry air was used as a reference, and differential refractivity of humid air was measured by varying the humidity of the air in the beam path over the range 0 to 62.5%, as determined by an ambient humidity meter, with results in Figure 1. The frequency resolution was 5 GHz.

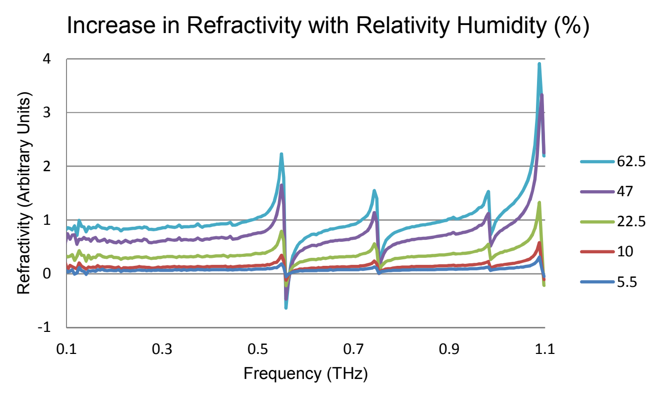

Fig. 1. Increase in refractivity as a function of frequency for five different values of relative humidity measured with a TDS.

Plots of the change in refractivity versus humidity at each frequency showed a linear relationship, with low data scatter, confirming the validity of the measurement, see Figure 2.

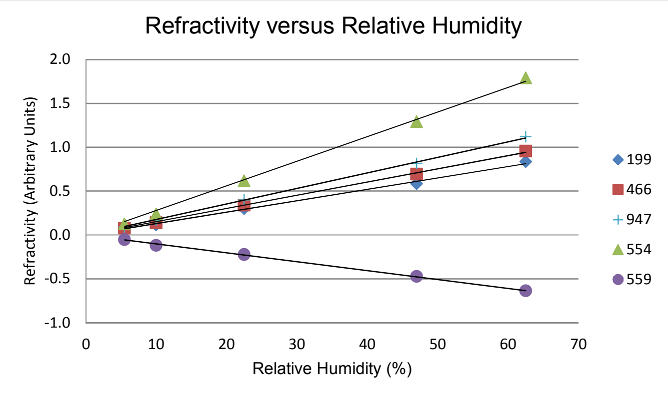

Fig. 2. The change in refractivity with relative humidity for five example frequencies, with fitted trend-lines super-imposed. The legend gives the frequencies in GHz. These are chosen to show a typical low-frequency value (199GHz), the general increase towards higher frequencies (496 and 947GHz) and the large positive and negative excursions on either side of the strong water-vapor transition at 557GHz. The relationship between refractivity and humidity remains linear in all cases, as expected.

The slope of the refractivity versus humidity relationship at each frequency was therefore taken to give a best estimate of frequency dependence of the refractivity due to water vapor. These measurements are not on an accurate absolute scale, since the true path through the instrument was not well determined. To compare the data with the models, we therefore took the ratio of the dispersive part to the non-dispersive part, the latter being taken to be the mean value in the range 220 to 270GHz. The refractivity at these relatively low frequencies is expected to approach the asymptotic "DC" value, which is reasonably well determined[3] and on which the three models being investigated here agree quite well.

To be specific the quantity plotted (see Figure 3) is

$$X(f) = [\, R(f) - Rm \,] / Rm,$$

where R(f) is the refractivity at frequency f and Rm is the mean refractivity between 220 and 270GHz. The same calculation was done for the measured data from the TDS and each of the models.

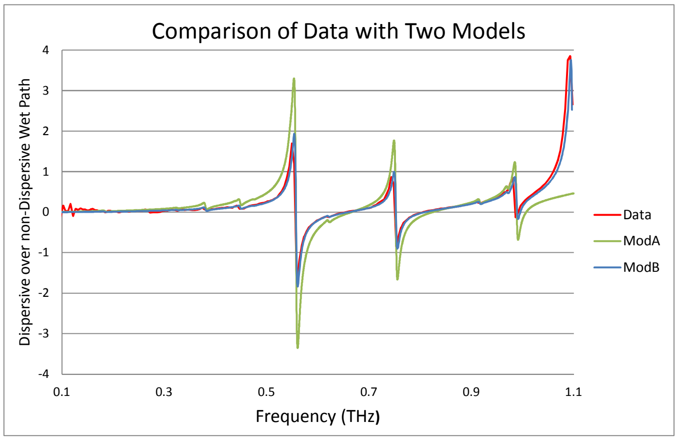

Fig. 3. The ratio of the dispersive component to the non-dispersive component as a function of frequency as determined from the measurements and from two of the models.

It can be seen that the values obtained from the laboratory data are in good agreement with one of the models, labelled ModB. This is in fact the "*am*" model developed by Paine[4]. It calculates the dispersive effect by first making a detailed estimate of the opacity over the whole spectral range from the radio right through to the far infrared and then performing a Hilbert transform. This transform is of the form derived from the Kramers-Kronig relationship between the real and imaginary parts of the complex refractive index. This approach has the advantage that models for the opacity have been have been well-tested both by experiment and by inter-comparison. Furthermore the Kramers-Kronig relations have a very firm theoretical basis. The method is however somewhat intensive computationally if the refractivity is only needed at one frequency.

The estimate from the other model (ModA) clearly disagrees with the data. In this case the discrepancies were traced to a coding error, plus the fact that the contributions from transitions at frequencies above 1THz were omitted. The third model (not shown) gave similar results to ModA. The cause in this case has not yet been found. It is not likely to be the result of the same error since the programs were written independently, even though both used the same general formulation, which is that described by Liebe[5]. Here the refractivity is found by calculating, at each frequency, the sum of the contributions from a restricted set of transitions at THz frequencies plus terms to take account of the remaining transitions at higher frequencies. This is quicker if the refractivity is only needed at one or a few frequencies. Note that the results given in the original reports by Liebe and colleagues do not appear to show the major discrepancies seen in the particular implementations that were available to us.

III. ACKNOWLEDGEMENT

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