Phase Locking of a Gyrotron Oscillator in the Hard Excitation Mode

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***Abstract*—In the theory of self-oscillating systems usually one distinguish two different kinds of self-excitation, namely soft and hard excitation. In the soft excitation mode, an unstable noise-level perturbation grows and evolves into a self-sustained oscillation. Conversely, in the hard excitation mode, a self-oscillation is sustained only for a sufficiently intensive initial perturbation with amplitude exceeding a certain threshold, while a small perturbation decays. In particular, in a gyrotron a maximal efficiency is often attained in the hard excitation regime. This work is aimed on study of injection-locked operation of a gyrotron in the hard excitation mode.**

I. Introduction

INJECTION-LOCKING of a gyrotron oscillator by an external driving signal provides output radiation frequency and phase stability, which is important for continuous-wave operation [1]. In addition, a low-power external driving may result in the fast frequency step tuning owing to the mode switching effect [2]. In a gyrotron, a maximal efficiency is often attained in the hard excitation regime [1]. This work studies injection-locked operation of two models of a gyrotron: the simplified quasi-linear model described by equations for slowly varying amplitude and phase [1,3], and the non-stationary model of a gyrotron with fixed Gaussian structure of the RF field [1].

II. Injection Locking of a Quasi-linear Model of a Gyrotron in the Hard Excitation Mode

At the first stage, we consider a simplified model described by the quasilinear equations

$$\begin{aligned} \dot{a} &= \left(\sigma + a^2 - a^4\right) a + f\cos\varphi, \\ \dot{\varphi} &= -\Omega + \beta a^2 - \frac{f}{a}\sin\varphi. \end{aligned} \tag{1}$$

Derivation of the basic equations of the quasi-linear theory of a gyrotron is presented in [1,3]. In (1), a and φ are slowly varying amplitude and phase, respectively, σ is the mode increment, β is the parameter of reactive nonlinearity, f is the driving signal amplitude, and Ω is the normalized detuning between the free-running gyrotron frequency and the driving signal frequency. To describe the effect of hard excitation, the fifth-order nonlinear term is retained in (1). Hard excitation in the free-running oscillator ($f = 0$) occurs at $-1/4 < \sigma < 0$.

The synchronized mode of oscillation is described by a steady-state solution of Eq. (1) with constant amplitude $a = a_0$, which satisfies the equation

$$\left[\left(\Omega - \beta M\right)^2 + \left(\sigma + M - M^2\right)\right] M = f^2 . \tag{2}$$

In Eq. (2) $M = a_0^2$.

Stability analysis of the steady-state solutions is performed.

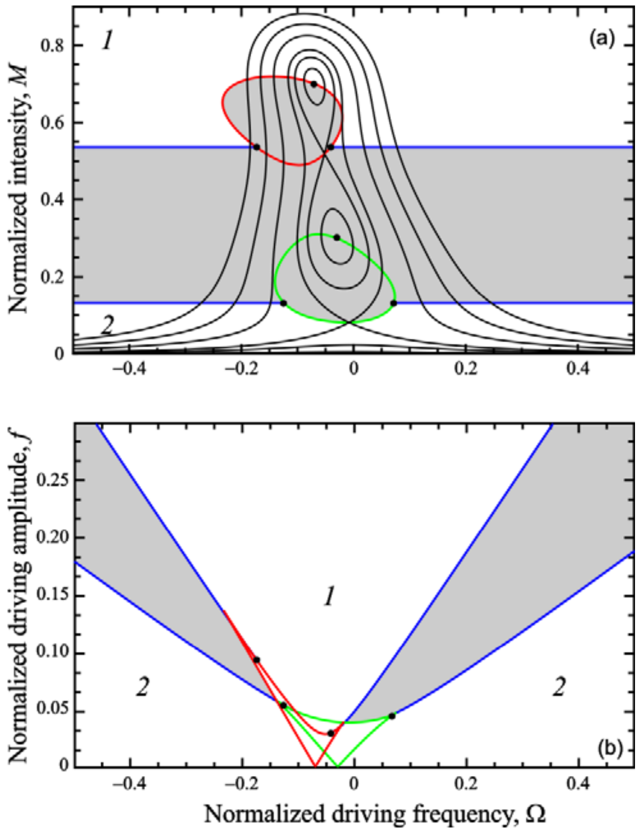

Fig. 1. Resonance curves (a) and phase diagram on the driving frequency – driving amplitude parameter plane for $\sigma = -0.21$, $\beta = -0.1$. *1* — phase-locking domain; *2* — regenerative amplification domain.

The results are in good agreement with the bifurcation analysis of the routes to synchronization given in [4]. Based on this analysis, we plot the resonance curves and phase diagrams on the (Ω,f) plane for different parameters. An example is presented in Fig. 1. On the contrary to the well-known pattern of synchronization for an oscillator with soft self-excitation [5], in Fig. 1(b) one can see two synchronization tongues corresponding to locking of the stable and unstable limit cycle, respectively. In a hard excitation mode, only a sufficiently large perturbation evolves into a stable periodic self-oscillation, while a perturbation with amplitude below a certain threshold decays [5]. Thus, the driven oscillator may operate as a regenerative amplifier of a small input signal. In Fig. 1(a), only the states above the upper bound correspond to the phase-locked self-oscillation (domain *1*), while the states below the lower one (domain *2*) correspond to the regenerative amplification. The domain of unstable steady states is shaded. There exist a small domain where bistability is observed, i.e., both injection locking and amplification regimes are stable.

III. Injection Locking of a Gyrotron with Fixed Gaussian Structure of the RF Field

At the second stage, the non-stationary model of a gyrotron with fixed Gaussian structure of the RF field [1] is considered. The basic equations are

$$\frac{dA}{d\tau}+A=iI\int_0^{\zeta_k}J(\zeta,\tau)f_s^*(\zeta)d\zeta+f_{inj}\exp(i\Omega\tau), \qquad (3)$$

$$\frac{dp}{d\zeta}+i\left(\Delta+|p|^2-1\right)p=iAf_s(\zeta). \qquad (4)$$

In Eqs. (3) and (4), p is the complex normalized electron orbital momentum, $\Delta=\left(2/\beta_\perp^2\right)\left(1-\omega_H/\omega_0\right)$ is the mismatch between the cavity mode eigenfrequency ω_0 and the cyclotron frequency ω_H, $\zeta=\left(\beta_\perp^2/2\beta_\parallel\right)\omega_0 z/c$ is the normalized axial distance, $\tau=\omega_0 t/2Q$ is the normalized time, I is the normalized current parameter, $\beta_\perp=v_\perp/c$, $\beta_\parallel=v_\parallel/c$, and

$$f_s(\zeta)=\exp\left[-3\left(\frac{2\zeta}{\zeta_k}-1\right)^2\right],\ 0\le\zeta\le\zeta_k$$

describes the axial Gaussian RF-field structure of the cavity mode.

The equations of motion (4) are integrated with the boundary condition

$$p(\zeta=0)=\exp(i\varphi_0),\ \ \varphi_0\in[0;2\pi). \qquad (5)$$

Form this integration one can find the harmonic of the bunched current

$$J=\frac{1}{2\pi}\int_0^{2\pi}pd\varphi_0\,. \qquad (6)$$

The electron orbital efficiency is

$$\eta=1-\frac{1}{2\pi}\int_0^{2\pi}\left|p(\zeta_k)\right|^2 d\varphi_0\,.$$

It is well known [1] that maximal transversal efficiency $\eta\approx 0.7$ is attained at $\mu=15$, $\Delta=0.53$, $I=0.06$ where $\mu=\zeta_k/\sqrt{3}$ is the normalized cavity length.

The phase diagram on the driving frequency–driving amplitude parameter plane is presented in Fig. 2(a). The pattern of the phase diagram is qualitatively similar to the quasi-linear model (1) (Fig. 1(b)). The domain of stable regenerative amplification is located below the line 2. The domain of phase locking of the gyrotron self-oscillation lies above the line 1. In Fig. 2(a), in the area where the phase-locking and regenerative amplification domains overlaps, i.e. below the line *1* but above the line *2*, there are two stable single frequency regimes. In this domain, the oscillator exhibits hysteresis as is shown in Fig. 2(b). The initial value of the amplitude should be taken large enough to achieve the high-amplitude mode.

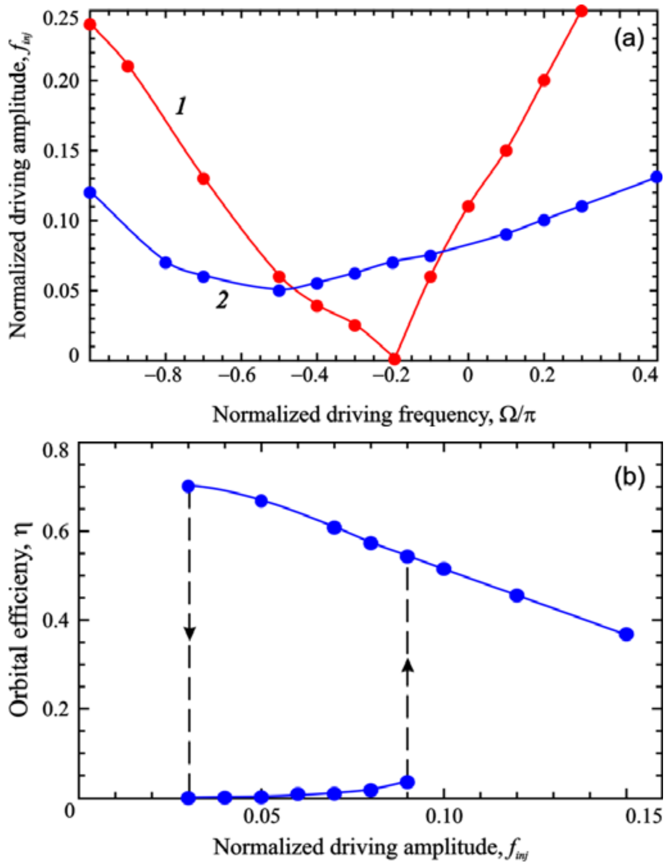

Fig. 2. (a) Bounds of injection locking (1) and regenerative amplification (2) on the $\Omega-f_{inj}$ plane for $I=0.06$, $\Delta=0.53$, and $\mu=15$. (b) Efficiency versus normalized injection amplitude at $\Omega=-0.3\pi$.

IV. Summary

The results of theoretical analysis of injection-locked operation of a gyrotron oscillator in the hard-excitation mode reveal several interesting features. Depending on the history of parameter values, the oscillator operates either as a regenerative amplifier of the input signal, or as an injection-locked self-oscillator. The results of theoretical analysis for the quasi-linear model (1) coincide with numerical simulation for the gyrotron with fixed Gaussian structure of the RF field.

V. Acknowledgment

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