Long-Distance Enhanced Fourier Transform by Hyperbolic Gradient-Index Metalens

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***Abstract*—A new hyperbolic gradient-index metalens is demonstrated at terahertz (THz) frequency, which can realize enhanced Fourier transform with extended spatial frequency bandwidth. The bandwidth can be extended to 2.5k_0 (k_0 is the wave-vector in air), which is 2 to 3 times of conventional lens. Furthermore, the working distance can be larger than one wavelength, and the input and output surfaces are flat, which provides conveniences for system integration. The metalens has potential applications on super-resolution imaging working over long distances.**

I. INTRODUCTION

As we all known, spatial Fourier transform (FT) is crucial in Fourier optics for applications such as performing mathematical operations [1] and spatial frequency filter for optical information processing [2]. When monochromatic plane wave transmits through a conventional plano-convex glass lens, the continuous changes in optical rays lead to spatial focusing, and different incident angles correspond to different focal positions. That is how spatial FT is done. Different types of meta-structures with the function of adjusting the effective index or the initial phase of emerging wave are proposed to realize plane gradient-index (GRIN) metalenses recently [3-5]. However, both the glass lens and the GRIN metalens has the limited spatial frequency bandwidths (corresponding to the numerical aperture (NA) of the lens). So the spatial FT can be accurate only up to a fraction of the wavenumber of the background medium. Even though the NA of a conventional lens can be enlarged simply by immersion technique [6], the high spatial spectrums corresponding to the evanescent incident waves can be detected only near the emerging interface (i.e. the near-field region) due to the lack of momentums compensation mechanism. To solve it, one can, of course, shape the surfaces of the immersion lens to be curve, but this is again not favorable for system integration. Besides, the increasing of the NA still remains modest due to the lack of available high index transparent materials. Recent reports proposed new ways to extend the spatial frequency bandwidth of lenses by transformation optics [7, 8]. However, they both have the curve input and output surfaces.

In this paper, we propose a new scheme to extend the spatial frequency bandwidth and the working distance of FT by a hyperbolic GRIN metalens at terahertz (THz) frequency. Due to the hyperbolic dispersion of the material and the momentums compensation in the material, evanescent waves with the transverse momentums up to 2.5k_0 (k_0 is the wave-vector in air) can be projected to the far-field. In this way, the bandwidth of the spatial FT can be extended to 2.5k_0.

II. RESULTS

In order to realize the transformation from evanescent waves to propagating waves, a 2D hyperbolic material with anisotropic permittivity (positive in one direction and negative in another direction) is employed. According to the dispersion equation of the material $k_y^2/\varepsilon_x + k_x^2/\varepsilon_y = k_0^2$, in which k_x, k_y are the longitudinal and transverse wave-vectors, respectively; and ε_x, ε_y are the relative permittivity in the corresponding directions, if $\varepsilon_x<0$ and $\varepsilon_y>0$, any evanescent wave in air ($k_y>k_0$) can be theoretically transformed into propagating wave in this kind of hyperbolic materials.

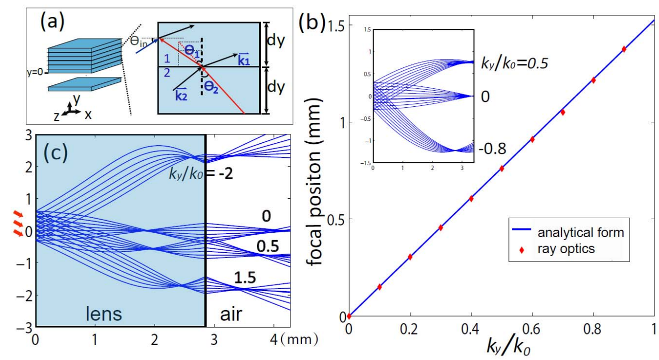

Fig. 1. Analysis and results of the ray tracing. (a) The model and light ray in the first two unit layer. The red solid lines with arrows represent the direction of energy flow and black solid lines with arrows represent the direction of phase velocity. (b) Comparison of the focal positions between the analytical form and the result of ray optics for the conventional isotropic GRIN lens. The schematic of focusing for k_y/k_0 = 0, 0.5, -0.8 are also present inset. (c) The trajectories of light rays for k_y/k_0 = 0, 0.5, 1.5, -2 in the hyperbolic GRIN lens. The working frequency is 1THz.

Now, let us consider the case of a hyperbolic-material-composed slab with GRIN permittivity. We assume ε_x = -1, ε_y keeps being positive and is the function of y. Because it is difficult to obtain the analytical expression of the fields in this kind of material, we divide the slab into numerous tiny layers with the thickness of every layer being dy, as shown in Fig. 1(a). If dy is much smaller than the thickness of the slab, each of the layers can be treated to be homogeneous. The incident and refraction angles (θ_1 and θ_2, respectively) of adjacent layers satisfy [3]

$$\frac{1}{\varepsilon_{y1}} - \frac{1}{\varepsilon_{y1}^2 \tan^2(\theta_1)} = \frac{1}{\varepsilon_{y2}} - \frac{1}{\varepsilon_{y2}^2 \tan^2(\theta_2)} \quad . \tag{1}$$

in which ε_{y1} and ε_{y2} are the y-direction permittivity in the first and second layer, respectively. If $\theta_1 = 90^\circ$, corresponding to the normal incidence ($\theta_{in} = 0^\circ$), we have $\tan^2(\theta_2) = \varepsilon_{y1}/[\varepsilon_{y2}(\varepsilon_{y1}-\varepsilon_{y2})]$. So only when $\varepsilon_{y1}>\varepsilon_{y2}$, the normal incident beam will converge in the dielectric core, otherwise it will diverge. Note that, θ_1 will also approach to 90° when total reflection occurs. In this case, we also wish the reflected beam will propagate toward to the axis of the lens for the purpose of focusing. So the same conclusion of $\varepsilon_{x1}>\varepsilon_{x2}$ can be also made. This conclusion directly suggests us the idea to design the hyperbolic GRIN lens. Different from the conventional isotropic GRIN lenses, the permittivity ε_y of the hyperbolic GRIN lens has the distribution of $\varepsilon_y = \varepsilon_c(1+ay^2)$, in which a is a positive constant, ε_c is the permittivity at $y = 0$ (i.e. the minimum permittivity).

Based on the above discussion, ray tracing is used to model the wave propagating trajectory in the lens. In order to check the validity of the theory, we first compare the analytical form of the focal position in the conventional isotropic GRIN lens (i.e. y_o=$2L_o k_y/[sqrt(\varepsilon_c)\pi k_0]$) with the result of the ray optics. As shown in Fig. 1(b), the focal positions in the y direction acquired from the ray optics are perfectly matched with the analytical form. The corresponding parameters are $\varepsilon_c = 2.01$, $L_o = 3.4$ mm, f_0=1THz. The schematic of focusing for $k_y/k_0 = 0$, 0.5, -0.8 are also present inset of the Fig. 1(b).

The trajectories of light rays for $k_y/k_0 = 0$, 0.5, 1.5, -2 in the hyperbolic GRIN lens are presented in Fig. 1(c), from which we can recognize the double focusing inside and outside the lens for both cases. The secondary focusing comes from the negative-refraction at the interface of the lens and air if the width of the lens is larger than the focal length. What should be pointed out is that, the cases of $k_y = 1.5k_0$ or $-2k_0$ correspond to the evanescent incident waves. It means that the hyperbolic GRIN lens can realize focusing for the evanescent waves in air over optically long distances as we have expected. So the input spatial frequency bandwidth can be theoretically extended by using the lens. The parameters of the lens are $\varepsilon_c = 1$, $a = 7.72e5$, and the width and the height are $L = 2.9$ mm and $W = 6$ mm, respectively.

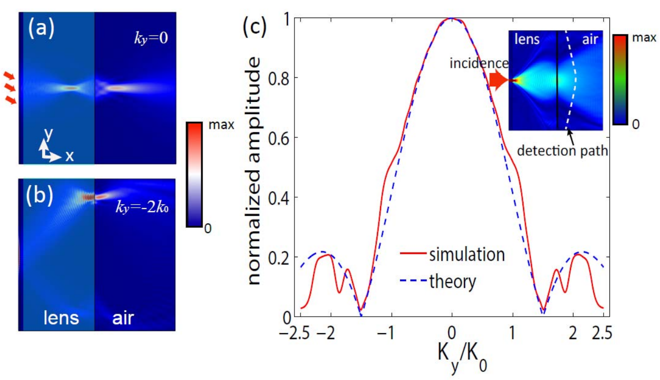

Fig. 2. Extraordinary focusing and spatial FT of the metalens. (a) and (b) present the double focusing for different incident case of k_y=0 and k_y= $-2k_0$, respectively. (c) presents the simulation (red line) and theory (blue dashed line) results of the spatial spectrum of a single rectangular pulse. The magnetic field pattern and the detection path are given inset. a=7.72e5

We further demonstrate our idea by the full-wave simulation, which is based on the Finite Element Method (FEM). As illustrated in Fig.2 (a) and 1(b), double focusing for the incident lights with k_y=0, $-2k_0$, respectively, are also clearly identified inside and outside the lens. The external focusing comes from the negative-refraction at the emerging interface. Note that, both the external positions are in the far-field region (larger than one wavelength). The length and width of the metalens are 20λ and 10λ, respectively, λ is the wavelength in air. The frequency is set at 1THz.

In order to verify the extraordinary spatial FT property of the metalens, we studied the case of a rectangular pulse incidence. The width of the pulse is 2λ/3. We detect the signal along the external curved focal plane, whose maximum and minimum distances apart from the emerging interface are 5λ and 1λ, respectively. The detected magnetic field amplitude is well matched with the theoretical result except for some small local differences, as illustrated in Fig. 1(c). The figure also directly indicates that the spatial frequency of the metalens can be extended to about $2.5k_0$.

A simple way to practically construct the metalens is to utilize the multi-layer hyperbolic metamaterials. One of the candidates to perform the negative permittivity material at THz frequency is the doped semiconductor InSb [9]. Note that, our proposed scheme can be effective not only in the THz frequency range but also in optics. Even though the input spatial frequency bandwidth of the metalens is just extended to $2.5k_0$, it can be further enhanced theoretically.

III. Summary

In summary, we have numerically demonstrated that the hyperbolic GRIN metalens can realize enhanced spatial FT with the spatial frequency bandwidth up to $2.5k_0$. Because the evanescent wave information can be acquired over optically long distances by the metalens, our work may find applications on long-distance super-resolution imaging.

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